Frequency Dependence of Poisson's Ratio of Viscoelastic Elastomer Foam

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SUMMARY

All polymer foams are viscoelastic; for foams used in earplugs, this response is essential to their function. Poisson's ratio was inferred from viscoelastic response of earplug foam in torsion and bending. Poisson's ratio increases with frequency as a result of stress induced air flow. Air within the pores contributes to the overall bulk modulus at high frequency, but not at low frequency at which it easily flows when the foam is deformed. Such behaviour is the opposite of that of solid polymers in which the shear modulus varies with frequency much more than the bulk modulus.

INTRODUCTION

Poisson's ratio in viscoelastic solids depends on time in the time domain or upon frequency in the frequency domain. The viscoelastic Poisson's ratio may be envisaged in the context of the following thought experiment. Does a stretched viscoelastic rod get fatter or thinner with time [1]? The transverse deformation of such a rod is described by the Poisson's ratio ν, which in viscoelastic materials depends on time or on frequency. In solid polymers, the viscoelastic Poisson's ratio increases with time; equivalently it decreases with frequency, owing to the effect of the glass transition upon the shear modulus in comparison with the bulk modulus. It has been suggested [2] that the time dependent Poisson's ratio $\nu(t)$ must be monotonically non-decreasing

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in all cases and that experimental results which indicate otherwise must be erroneous by virtue of the theory of viscoelasticity. This is certainly sensible for solid polymers in which the shear modulus decreases by orders of magnitude with increasing time or decreasing frequency, while the bulk modulus changes little; for isotropic materials, Poisson's ratio depends on the shear and bulk moduli as discussed in detail below. Examples to the contrary have been presented in materials which support coupled fields [3]. For example one may envisage a negative Poisson's ratio elastic foam [4] skeleton containing in the interstices a viscoelastic foam of smaller cell size (microcellular) with a conventional cell structure [5]. For a short time or a high frequency, the small cell viscoelastic foam is assumed to be sufficiently stiff that it provides most of the stiffness. The short time Poisson's ratio therefore approximates that (about 1/3) of a conventional foam. For a long time or a low frequency, assume that the modulus of the small cell viscoelastic foam relaxes to zero. Then the material has properties equal to those of the negative Poisson's ratio foam skeleton. The Poisson's ratio of this cellular solid therefore decreases with time.

The viscoelastic Poisson's ratio has a different time dependence depending on the test modality chosen; it may appear different in creep and relaxation. The difference, for a moderate degree of viscoelasticity, is minor. Interpretation of the dynamic Poisson's ratio in the frequency domain is unambiguous. The dynamic frequency domain approach has the further advantage of simplicity [6] in that viscoelastic results can be obtained from elastic equations using the dynamic correspondence principle by replacing elastic constants with complex moduli.

To realize a material in which Poisson's ratio increases with frequency, consider a porous material with a viscous fluid in the interstices. If the material is a polymeric open cell foam, one expects the time or frequency dependence of its viscoelastic behaviour to be the same as that of the polymer making up the ribs [7]. Moreover, in such foam, the frequency dependence will be the same in torsion or bending. That is the case if the deformation is sufficiently slow that any fluid (air or water) in the pores has sufficient time to move easily in response to stress. Stress-induced fluid flow in porous media gives rise to time-dependent behaviour as analyzed by Biot. This viscoelasticity depends on a volume change to move the fluid. By contrast, shape changes in shear give rise to no viscoelasticity due to fluid flow, though in a polymer matrix there will be viscoelasticity associated with the polymer itself. Since the effective bulk modulus of the fluid-filled cellular polymer decreases with time due to stress-induced movement of the fluid, the Poisson's ratio also must decrease with time, or increase with frequency. In an isotropic material, the moduli and Poisson's ratio are related and that relationship allows one to infer Poisson's ratio from the moduli. Stress-induced airflow in the open cell foam gives rise to macroscopic viscoelastic effects. Creep due to such a mechanism via Biot theory [8] predominantly follows a single exponential over a range of time; in the frequency domain the corresponding dependence of storage modulus E' on angular frequency ω follows the Debye form:

$$E' = E_0 + E_1\, \omega^2\tau^2 / (1 + \omega^2\tau^2) \qquad (1)$$

The time constant τ is inversely proportional to the permeability and is proportional to the square of the distance over which the fluid flows. Such effects are known in foams [9]. Air at atmospheric pressure (100 kPa) contributes an effect in Young's modulus on the order of 100 kPa. Such an increment in modulus is expected at a time scale or a frequency scale associated with the stress induced fluid flow. In foam, the air phase contributes to stiffness at short time scales or high frequencies. The air does not provide stiffness in foam (with communicating porosity between cells) at long time or low frequencies since it is then free to escape through the pores.

Foam earplugs are suitable for studies of stress-induced airflow because the foam is compliant and the cell size is relatively small. Viscoelasticity is intentionally introduced into this kind of earplug, which is larger than the ear canal. The user rolls the plug into a narrow rod. The foam of which the plug is made retains its deformed shape for sufficient time so that the user can insert it into the ear. The US patent [10] for the Classic AEREO safety earplug foams studied previously, prescribes that the compressed plugs are intended to recover their initial compression in about 2–20 s giving the user enough time to insert it into the ear. Following insertion, the foam then expands in creep recovery to slowly fill the ear canal, excluding noise.

In the present study, viscoelastic foam from earplugs was characterized dynamically in torsion and bending to infer the frequency dependent Poisson's ratio.

MATERIALS AND METHODS

Measurements of viscoelastic moduli in torsion and bending were conducted at ambient temperature, 22°C, using broadband viscoelastic spectroscopy [11] (**Figure 1**). Torque (sinusoidal for dynamic studies) was produced electromagnetically by electric current in a Helmholtz coil acting upon a high intensity neodymium iron boron magnet at the specimen free end. Angular displacement was measured via laser light reflected from a small mirror upon the magnet to a position-sensitive silicon light detector. To perform torsion and bending experiments, an electric current was input to the appropriate coil and the corresponding detector axis orientation was used. There are two orthogonal

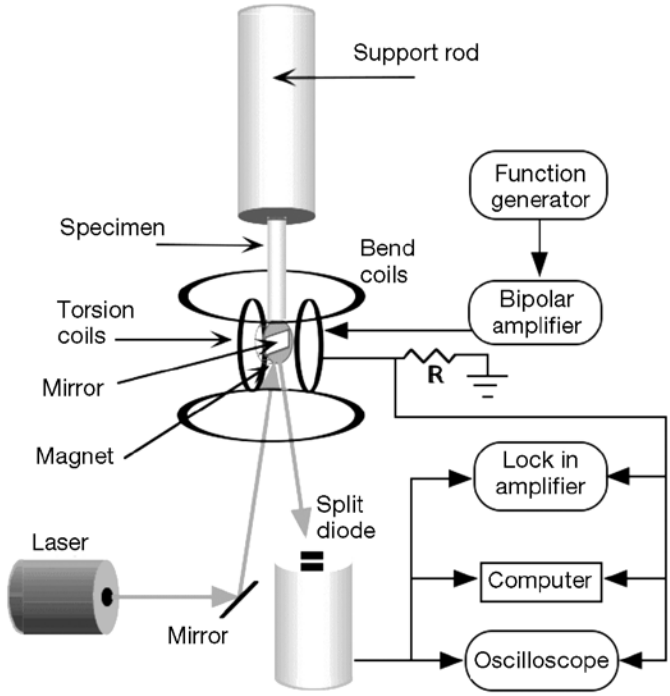

***Figure 1**. Broadband viscoelastic spectroscopy apparatus*

coils, one for torsion and one for bending. The magnet was calibrated for torque using slender specimens of the well-known 6061 aluminum alloy. The light detector was calibrated using a precision micrometer. Signals proportional to torque and angular displacement were input to a lock in amplifier (SRS type SR 850) to determine magnitude; phase angle was not considered in this study. The magnitude ratio (a structural stiffness) between torque and angle are proportional to the magnitude ratio between stress and strain (a material stiffness) at frequencies well below resonance. One can correct for approach to resonance provided the geometry of specimen and attached inertia is sufficiently simple. The data point at the resonant frequency, the highest one plotted, was obtained using analysis of a lumped resonating system with one degree of freedom. The mass moment of inertia of the attached end piece, magnet and of the foam itself were input to the calculation.

Viscoelastic open cell foam was cut into a cylindrical disc shape 7 mm in diameter from earplugs (Mack's safe sound, Mkeon Products, Warren, MI) and was cemented on one side to a cylindrical support rod, and on the other side to a polymer (PMMA) end piece 0.5 mm thick. The end piece covered the entire end of the specimen. The magnet providing the driving torque was a light weight cube about 1 mm on a side glued to the end piece. This earplug foam (a polyurethane foam) has a relatively small cell size similar to that of earplug foam studied previously [12].

Poisson's ratio was inferred from frequency-dependent torsional and bending moduli. Poisson's ratio is given in terms of shear modulus G and Young's modulus E for isotropic materials by:

$$E = 2G\,(1 + \nu) \qquad (2)$$

In tests of stiff materials, it is practicable to use the slender long rod geometry required to easily infer E from bending. At the other geometrical extreme, for an extremely short specimen approximating a thin disc, the modulus is the tensorial constrained modulus [13] related to Poisson's ratio for isotropic materials as:

$$C_{1111} = 2G\,[\nu/(1-2\nu) + 1] \qquad (3)$$

For $\nu = 0.3$, the observed modulus of such a short specimen is 1.35 E. The reason is that the stiff constraints on the ends restrain the Poisson effect so that the observed stiffness is greater than Young's modulus. For compliant material such as polymer foam, a slender rod specimen geometry is problematical because such a rod has a low natural frequency, which restricts the available frequency range. Therefore short specimens were used with thickness h smaller than the radius r. Results from short specimens down to radius to thickness ratio r/h of 3 can be interpreted by approximate or numerical means. For example [14] for r/h = 3 and Poisson's ratio $\nu = 0.3$, the observed modulus in compression is about 7% greater than the Young's modulus. In the present study, specimens of diameter 7 mm and thickness 3 mm were interpreted using this method.

RESULTS AND DISCUSSION

An image of typical foam structure is shown in **Figure 2**. The structure is independent of direction as revealed by observation of multiple sections indicating isotropic physical properties. Cells were mostly from 0.1 mm to 0.4 mm in diameter; there was communicating porosity between cells sufficient that the foam could be squeezed flat with little effort. The foam sample had a density 0.29 g/cm^3 based on mass and volume. Maximum surface strain during the tests was about 10^{-4}. Results of viscoelastic measurements are shown in **Figures 3** and **4**. The magnitude of complex moduli increases with frequency as expected for a viscoelastic elastomer. The moduli at low frequency are reasonable for a flexible foam rubber material.

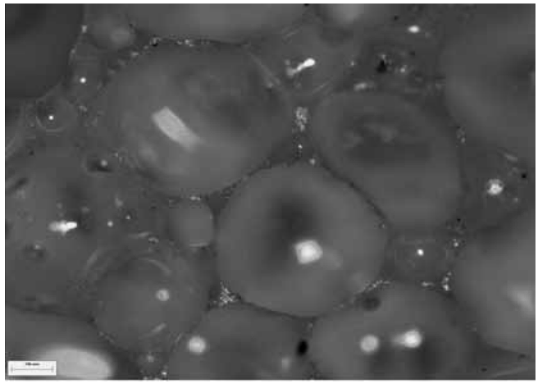

Figure 2. *Image of foam structure. Scale bar, 100 microns*

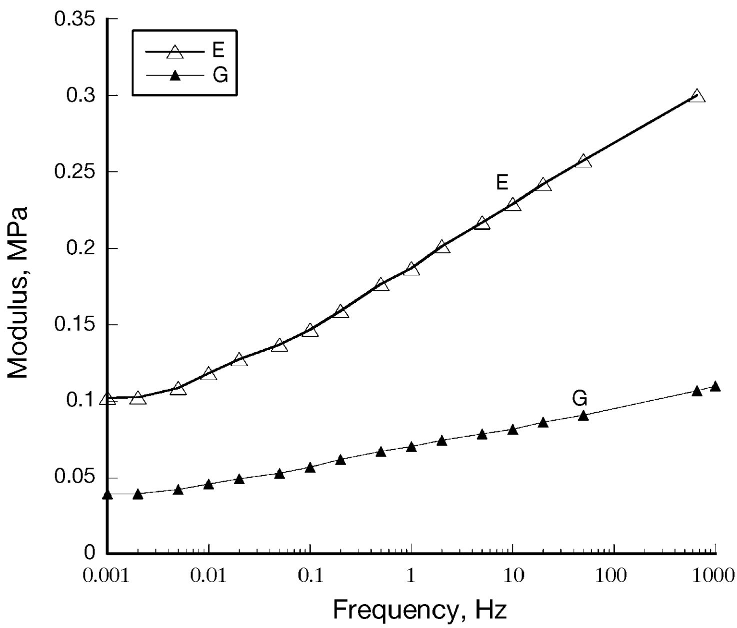

Figure 3. *Viscoelastic response: absolute value of Young's modulus, |E*| and shear modulus |G*| vs. frequency*

The Poisson's ratio was observed to increase with frequency as shown in **Figure 4**. As discussed above, elastic properties of foam in the absence of fluid-solid interaction are governed by the Gibson Ashby relations:

$$E_{foam} / E_{solid} = [\rho_{foam} / \rho_{solid}]^2, \; G_{foam} / E_{solid} = (3/8)[\rho_{foam} / \rho_{solid}]^2 \quad (4)$$

In viscoelastic materials, by the correspondence principle, the moduli become complex and frequency dependent. In the following, the star * indicates a complex quantity with magnitude and phase.

$$E^*_{foam} / E^*_{solid} = [\rho_{foam} / \rho_{solid}]^2, \; G^*_{foam} / E^*_{solid} = (3/8)[\rho_{foam} / \rho_{solid}]^2 \quad (5)$$

Because the elastic Poisson's ratio is given by:

$$\nu = E/2G - 1$$

the viscoelastic Poisson's ratio is:

$$\nu^* = E^*/2G^* - 1 \quad (6)$$

So, for a foam in which the moduli depend solely on bending of cell ribs (without fluid solid interaction), any frequency dependence and phase angles divide out, so that the Poisson's ratio is a real quantity without phase or frequency dependence, a function only of the structure of the foam. For most isotropic foams, the Poisson's ratio is close to 0.3. Indeed, such a Poisson's ratio is determined at low frequencies in the present results. The curve fit in **Figure 4**

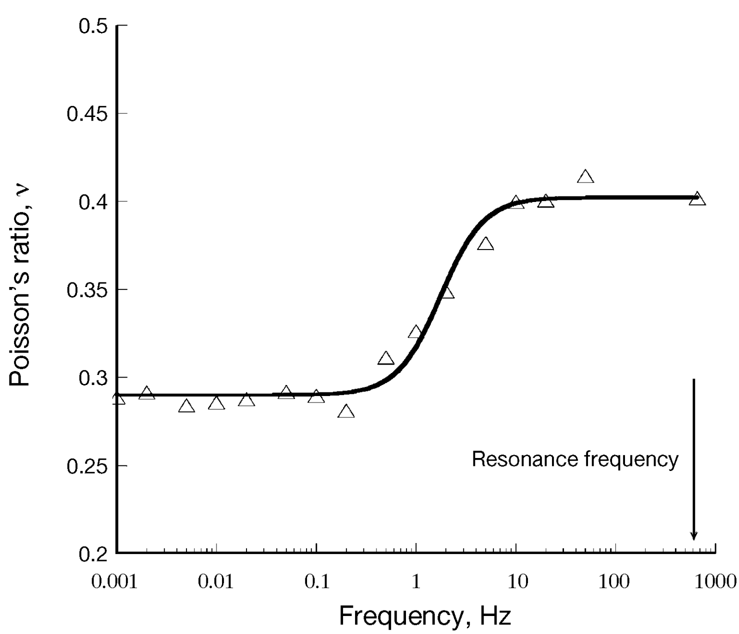

Figure 4. *Viscoelastic response: Poisson's ratio ν vs. frequency. The curve fit is based on $\nu = \nu_0 + \nu_1 \omega^2\tau^2 / (1 + \omega^2\tau^2)$ with ω as (2π)(frequency) as described in the text*

is based on a Debye form (Equation (1)) for Poisson's ratio, shown in the caption, with a time constant τ = 0.56 s, initial Poisson's ratio ν_0 = 0.29 and increment in Poisson's ratio ν_1 = 0.112. We remark that the time constant will differ under the large strain conditions associated with the use of the foam in earplugs; large compressive strain will likely slow the flow considerably.

The observed frequency dependence of the Poisson's ratio is interpreted in the context of stress-induced flow of air in the pores. At the higher frequencies, the modulus in bending increases more than the modulus in torsion because bending entails a change in volume during deformation. The volume change drives flow of air at the lower frequencies and compresses air in the pores at higher frequencies. Poisson's ratio by virtue of its relation to the moduli therefore increases with frequency in this regime. At sufficiently high frequencies above those applied here, moduli are expected to become so high that the increment in axial stiffness due to entrained air becomes negligible. In that regime, Poisson's ratio is expected to decrease back to about 0.3. The frequency dependence of Poisson's ratio of flexible foam differs from that of solid polymers because the airflow mechanism gives rise to an increase in bulk modulus hence Young's modulus and Poisson's ratio with frequency. By contrast in solid polymers molecular rearrangement has much more effect on the shear modulus than on the bulk modulus.

CONCLUSIONS

Poisson's ratio of viscoelastic earplug foam increases with frequency as a result of stress-induced airflow. Such behaviour is the opposite of that of solid polymers and it is in contrast to suggestions made by some authors that Poisson's ratio must increase with time (decrease with frequency) for *all* materials. The difference occurs for the following reasons. For the foam, air within the pores contributes to the bulk modulus at frequencies sufficiently high that there is insufficient time for it to escape or move to a region of lower applied strain. Poisson's ratio by virtue of its relation to the moduli, increases with frequency in these foams.

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