

**Kalathur, H. and Lakes, R. S., "Enhancement in piezoelectric sensitivity via negative structural stiffness" *Journal of Intelligent Material Systems and Structures*, 27(18) 2568-2573 (2016): preprint version**

**Abstract**

Effective piezoelectric sensitivity of bimorph strip actuators was enhanced by negative structural stiffness. Negative stiffness was achieved in brass strips post-buckled in compression to an 'S' shape; the brass strip was placed in series with the bimorph. The negative stiffness was tuned by adjusting the brass strip length. The effective piezoelectric sensitivity in units of displacement per volt input, increased as the inverse of the overall stiffness of the series element. The maximum observed enhancement in effective piezoelectric sensitivity was at least a factor of six, at 1 Hz in comparison to the value when no negative stiffness was used. This corresponds to a maximum effective sensitivity of about 36  $\mu\text{m}/\text{V}$  in comparison to the baseline effective sensitivity of about 6.1  $\mu\text{m}/\text{V}$ , at 1 Hz. The value is several orders of magnitude (almost 40,000 times) higher than the typical value of sensitivity for the material comprising the individual bimorph strip actuator.

*Keywords:* Piezoelectric, Negative Stiffness, Enhancement, Sensitivity

*Abbreviations:*

APC American Piezo Ceramics  
MTI Mechanical Technology Incorporated  
SRS Stanford Research Systems  
LVDT Linear Variable Differential Transformer

*Units:* micrometer ( $\mu\text{m}$ ), millimeter (mm), Newton per millimeter (N/mm), micrometer per volt ( $\mu\text{m}/\text{V}$ ), volts (V), Hertz (Hz)

**1. Introduction**

Piezoelectricity is a property exhibited by certain classes of crystalline materials, wherein stress causes an electric polarization in these materials. Conversely, an electric input to these materials gives rise to mechanical strain. (e. g. [1]). Piezoelectricity is a coupled field phenomenon that requires the material to lack a center of inversion symmetry. This very property renders such materials useful in electromechanical transducers. Piezoelectric ceramics typically exhibit sensitivity values that are on the order of nanometers per volt in systems in which the polarization causes bending and on the order of 30-500 picometers per volt in systems in which the polarization causes axial deformation. This property enables these materials to be used as actuators depending on the application. For example, if the application warrants more deformation and less force, benders may be used and if the requirement is more force and less deformation, then axial deformation actuators, or stacks of axial actuators, may be used. Piezoelectric materials are always anisotropic (they must be chiral) and adhere to the following constitutive equations [1],

$$\boldsymbol{\varepsilon}_{ij} = J_{ijkl} \boldsymbol{\sigma}_{kl} + d_{kij} \mathcal{E}_k + \alpha_{ij}\Delta T \quad (1)$$

$$D_i = d_{ijk}\sigma_{jk} + \kappa_{ij}\mathcal{E}_j + p_i\Delta T \quad (2)$$

The strain tensor  $\boldsymbol{\varepsilon}_{ij}$ , depends on the stress tensor  $\boldsymbol{\sigma}_{kl}$  via the elastic compliance, the electric field  $\mathcal{E}_k$  via the modulus tensor  $d_{ijk}$  and the change in temperature via the thermal expansion tensor  $\alpha_{ij}$ . In addition,

the electric displacement vector depends on the electric field via the dielectric tensor  $K_{ij}$  at constant stress and the temperature change via the pyroelectric coefficient  $p_i$  also at constant stress.

Negative stiffness, in contrast to positive stiffness is a phenomenon in which the restoring force is in the same direction as the applied displacement. Negative stiffness can be achieved [2] in many ways such as a post-buckled ‘S’ shaped slender bar, which is in unstable equilibrium. Another paradigm is a composite with particulate inclusions wherein, when the temperature approaches the phase transformation of the inclusions, the result is an induced negative stiffness of these inclusions that is constrained by the surrounding matrix material. Negative stiffness typically is considered non-conservative if it occurs in the post-buckling regime in a structural load bearing context. Negative stiffness elements such as a buckled tube can be stabilized by a stiff constraint; when appropriately tuned, large viscoelastic damping and controllable stiffness were attained [2a]. The theoretical basis is that of a one dimensional Reuss composite with complex moduli. A Reuss composite is one wherein the constituent layers are stacked on top of one another and the loading is in the transverse direction with respect to the layers. Consequently, all the layers are subject to the same load but deform differently. The Reuss formula for continuous media is

$$\frac{1}{E_c} = \frac{V_1}{E_1} + \frac{V_2}{E_2}$$

in which  $E_c$ ,  $E_1$  and  $E_2$  refer to the Young's modulus (stiffness) of the composite, phase 1 and phase 2, and  $V_1$  and  $V_2$  refer to the volume fraction of phase 1 and phase 2 with  $V_1+V_2=1$ .

More recently, potentially practical dampers with enhanced viscoelastic damping capacity have been developed based on constrained negative stiffness [3], [4], [5].

Piezoelectric materials can be used in a layered structure, a two layer composite paradigm, to attain enhancement in the overall response of the composite. Piezoelectric coefficients [6] have been derived for these types of composites using the Voigt and Reuss type formulations. The piezoelectric coefficient such as  $d_{33}$ , has been shown to follow a rule of mixture form. Further, a piezocomposite [7] wherein one of the phases consists of a piezoelectric ceramic and the other phase is a passive polymer phase with negative Poisson's ratio has been presented. The model was shown to establish enhanced electromechanical coupling in the piezocomposite. Particulate composites [8] with negative stiffness inclusions allow for extreme piezoelectric and pyroelectric properties in addition to high thermal expansion that exceeded conventional bounds. Doped ceramics [9], [10] and [11] have been synthesized and shown to exhibit improved dielectric, piezoelectric and electromechanical coupling properties as well as the poling efficiency. Piezoelectric actuators have also been shown to be very useful in vibration control [12] because they allow for compactness of design, and in vibration damping such as in plate structures on space vehicles [13].

Previous studies relevant to this paper also include the effect of dimensions of a piezoelectric unimorph cantilever and the resonance mode [14], both theoretically and experimentally, on the mass detection sensitivity of the cantilever. The mass at the cantilever tip coupled with the change in dimensions and the resonance mode of the cantilever caused an increase in the resonance frequency shift per unit mass by a factor of  $\alpha^4$  where  $\alpha$  is the proportional reduction in size of the piezoelectric unimorph cantilever. Enhancement in piezoelectric sensitivity is pertinent in the context of actuators where relatively large deformations are necessary. Lattice structures [15] with bend-dominated rib elements have been developed and the effective sensitivity was shown to exceed the intrinsic sensitivity of the material comprising individual ribs by several orders of magnitude. In addition, a fully triangulated lattice with curved ribs [16], which exhibits a stretch dominated response for a small rib curvature, is also shown to exhibit arbitrarily high piezoelectric sensitivity that is unbounded.

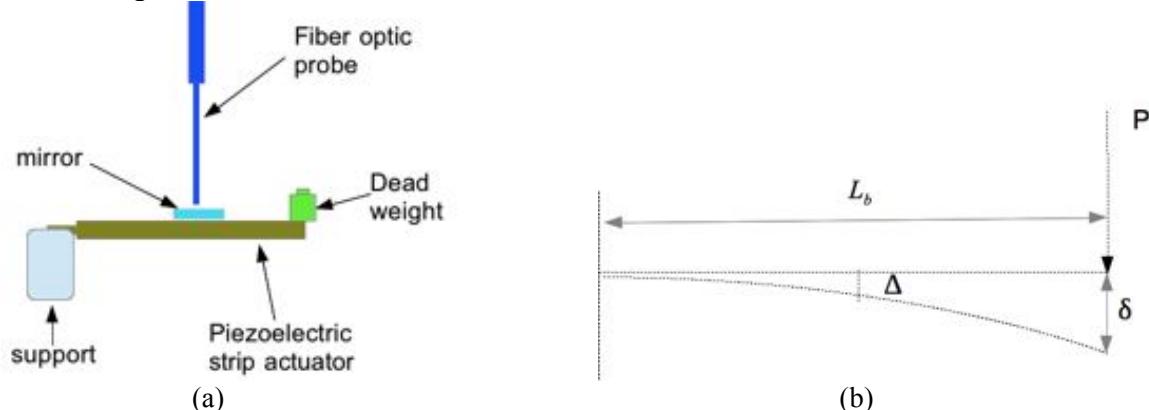
In this experiment, a piezo mechanical hybrid paradigm has been employed to achieve an enhancement in the effective piezoelectric sensitivity. A bimorph actuator cantilever was used in conjunction with a post-buckled brass strip, which is non-piezoelectric and entails negative stiffness. The brass strip was deformed by axial buckling load into an ‘S’ shape in the linear range of the material, and connected in

series with the piezoelectric actuator with positive bend stiffness. The negative stiffness was tuned to approach the positive bend stiffness of the actuator. Two brass strips of different thicknesses and widths were cut out from two different brass sheet stocks. The thicker and wider strip had to be used to avoid excessive buckling while tuning the negative stiffness.

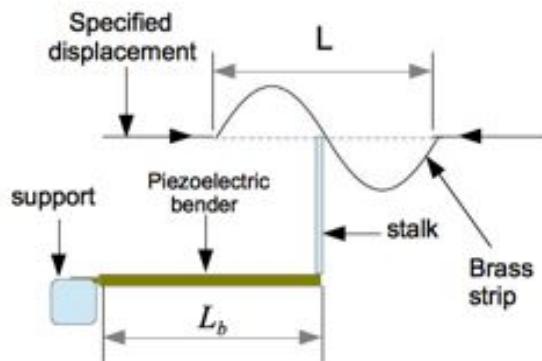
## 2. Methods

For all measurements, apparatus was mounted on an optical breadboard (Thorlabs, Inc., NJ) supported by viscoelastic rubber mounts to reduce vibration transmission from the building. The bend stiffness of the piezoelectric strip actuator (APC international, Ltd., PA) cantilever was first measured with the help of a known weight of 2g at the tip (figure 1a). The deflection,  $\Delta$  at the center of the cantilever was then measured using a fiber optic displacement sensor (MTI 2000 fotonics sensor) and the obtained value was used to infer the tip deflection  $\delta$ , using the usual cubic dependence

$$\left[ \delta_x = \left( \frac{Px^2}{6EI} \right) (3L - x), \Delta = \frac{5PL^3}{48EI}, \delta = \frac{PL^3}{3EI} \Rightarrow \delta = 3.2\Delta \right]$$
 of lateral deflection with respect to the beam length. The bend stiffness was then calculated by dividing the known weight ( $\approx 0.02$  N) by the tip deflection,  $P/\delta = \frac{3EI}{L_b^3} \approx 4.8 \times 10^{-2} \text{ N/mm}^3$ . This value was used as the baseline to tune the negative stiffness of the brass strip. The concept of sensitivity enhancement with negative stiffness is shown in figure 2.



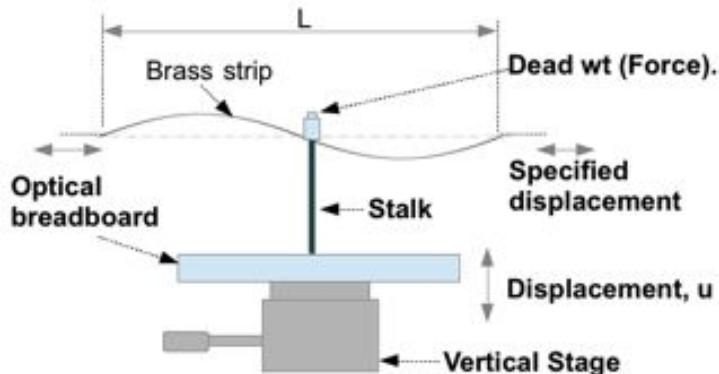
**Figure 1:** (a) Measurement set up for the determination of the bend stiffness of the piezoelectric actuator cantilever. The cantilever length  $L_b$  is not shown here. (b) Determination of the tip deflection using the usual cubic dependence.  $\Delta$  is the center deflection determined using the fiber optic sensor and the fiber optic probe.



**Figure 2:** Piezoelectric sensitivity with negative stiffness paradigm. The curvature of the buckled brass strip is exaggerated by a factor of about 100.

Brass strips of widths of about 15mm and 30mm, and 0.4mm thick were cut from a sheet. The brass strip was secured at the ends with the help of grips. The negative stiffness was measured for each chord length ‘L’ using a dead weight method, which is described as follows. First, a shallow ‘S’ shape was generated by adjustment of micrometer driven horizontal stages (Newport 460A series) at a chord length of  $L = 195\text{mm}$ . A micrometer driven vertical stage (Newport MVN50) was adjusted so that the bi-stable brass strip rests on the stalk under no dead weight. Once stabilized, the vertical stage was moved using the micrometer until the brass strip snapped upwards. The corresponding vertical displacement was recorded. This served as the reference point. The micrometer was brought back to the original position. Now, a known weight of e.g. 0.005N (0.5g) was placed (figure 3) above the brass strip at the center. The micrometer was again turned until the brass strip snapped upwards and the corresponding vertical displacement was recorded. The vertical stage was brought to the original position. This procedure is repeated for several dead weights. Finally, the relationship between dead weights and the vertical displacements corresponding to the snapping up was plotted. The negative stiffness was determined from a linear fit of the resultant plot. For the sake of clarity, the force and displacement are assumed positive upward. Contrary to the usual relationship in which, when both the force and displacement are in the same direction, the result is positive stiffness. In this case, the buckled brass strip entails negative stiffness and hence the directional relationship is flipped so that when the force increases, the displacement decreases.

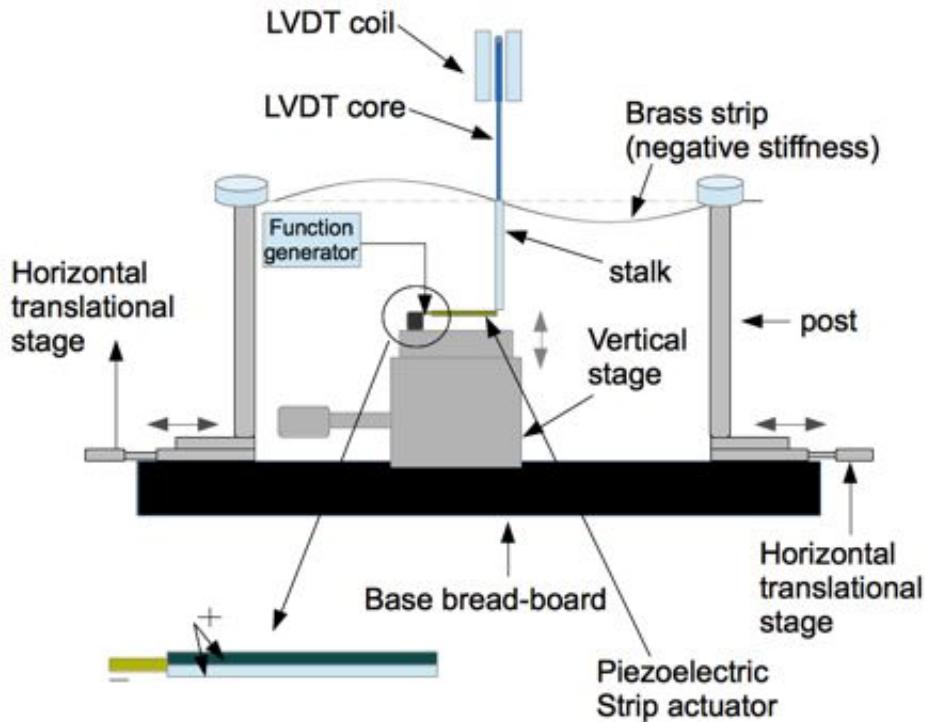
Now, the chord length was changed uniformly at both ends of the brass strip, with the help of micrometer driven horizontal stages (Newport 460A series). The same procedure as above was repeated for determining the negative stiffness for the new chord length ‘L’. The mid-deflection of the brass strip that is representative of the amount of negative stiffness, (‘S’ curve) was measured using a caliper.



**Figure 3:** Negative stiffness measurement set up.

The effective piezoelectric sensitivity in this experiment is defined as the peak-to-peak value of displacement divided by the peak-to-peak value of the input voltage applied to the piezoelectric actuator. The displacement was measured using an LVDT (Trans Tek 240-000). First, the sensitivity of an individual piezoelectric strip (APC international cat. No. 40-1035) actuator as a cantilever without negative stiffness was measured. (The piezoelectric strip had dimensions 49 mm long by 1.8 mm wide by 0.6 mm thick). The result was used as the baseline. The piezoelectric sensitivity enhancement was achieved using the set up shown in figure 4. In addition, an elastic rubber band (not shown in figures) was used around the two posts to prevent the horizontal stages from sliding apart against their built-in springs when the brass strip is buckled. The required LVDT input of 24V was provided using a Matsuada dc power supply. An ac input voltage to the bimorph strip of 21V peak to peak at 1Hz, was provided via a SRS type DS345 function generator amplified by an Avtech amplifier. The output displacement signal from the LVDT was run through a low pass filter (Stanford Research Systems, SR560, cut off frequency = 10 Hz) and monitored using a digital oscilloscope (Tektronix TDS 3014B). The input drive signal was also monitored on the scope. A parallel output signal from the filter was fed into a SR850 lock-in

amplifier to measure the precise rms displacement value, which was then multiplied by  $2\sqrt{2}$  to obtain the peak-to-peak value. The scope was used to monitor the linearity of the signal. The effective piezoelectric sensitivity associated with the composite negative stiffness system was determined for each chord length of the brass strip. The chord length was tuned by micrometer adjustment of the two horizontal stages.



**Figure 4:** Enhancement of effective piezoelectric sensitivity using negative stiffness.

### 3. Results and discussion

Negative stiffness inferred from linear fit of load deformation curves for a shallower ‘S’ curve brass strip follows a linear form, as shown in figure 5a. As the ‘S’ curve becomes steeper, deviation of the data from a straight line was seen, due to structural non-linearity. Figure 5b which shows negative stiffness versus chord length change, is plotted using the same data set as figure 5a. The error bars shown are based on the maximum deviation (which is about 10%) of the negative stiffness value from a value that yields a linear correlation with enhanced sensitivity. This is reasonable because the accuracy of negative stiffness measurement is easily within 10%, at steeper ‘S’ curves.

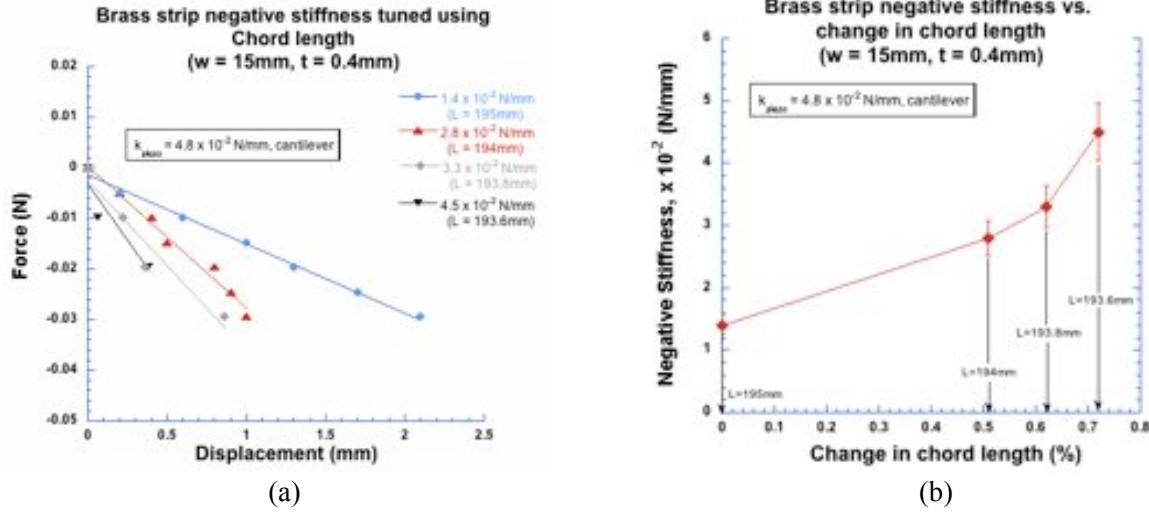
The effective piezoelectric sensitivity defined as the peak-to-peak displacement output per unit peak-to-peak volt input, increased as the inverse of the overall stiffness of the series element. The solid triangular points in figure 6 show results for the 15 mm wide brass strip. The maximum piezoelectric sensitivity enhancement was about a factor of three. The change in the chord lengths,  $\Delta L/L$ , for each of these triangular points were about 0.00%, 0.51%, 0.62% and 0.72% respectively in figure 6, with respect to  $L = 195\text{mm}$ , for  $w = 15\text{mm}$ . Progressive tuning led to a steeper ‘S’ curve in the brass strip, giving rise to nonlinearity, marginal stability and difficulty in fine tuning. Therefore a wider 30 mm wide brass strip was used, which allowed for a shallower ‘S’ shape. A solid circular point in figure 6 shows the result for the 30 mm wide brass strip. For this case,  $\Delta L/L = 0.3\%$  with respect to  $L = 244\text{mm}$ , the corresponding length of that strip. The maximum piezoelectric sensitivity enhancement was about a factor of six.

The baseline effective sensitivity of the piezoelectric bender without negative stiffness was about  $S_{\text{without}} = 6.1 \mu\text{m/V}$ , at 1 Hz. This was enhanced by a factor of about six to,  $S_{\text{with}} = 36 \mu\text{m/V}$ , at 1Hz, with the  $w = 30\text{mm}$  brass strip (figure 6). The present study emphasizes linear response, however snap-through buckling can also give rise to nonlinear hysteresis which may be useful in some applications.

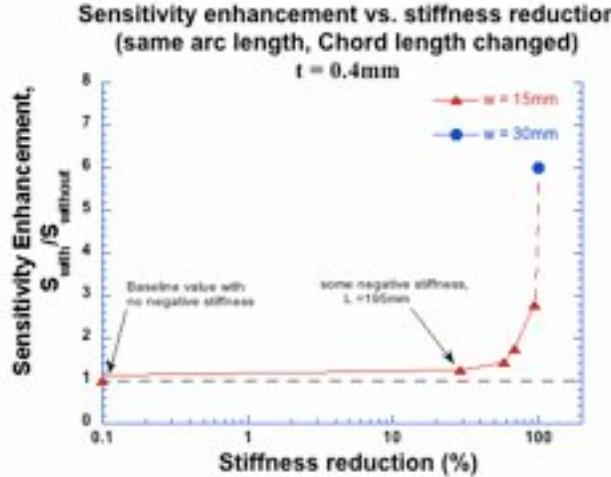
Piezoelectric bender actuators are used in switches, manufacturing machines, pressure regulators, and valves. If performance is at a premium, enhanced actuators such as those studied here may be considered.

It is also possible to enhance effective performance of piezoelectric elements by adding an external electric circuit. In some such approaches, negative capacitance is achieved using a feedback circuit which, in contrast to the present approach, uses an external source of power.

In summary, enhancement of piezoelectric sensitivity by a series elastic element of negative stiffness in theory tends to infinity as the negative stiffness is tuned to match the positive stiffness of the piezoelectric actuator element. Experimentally, the effective piezoelectric sensitivity indeed increased in inverse proportion to the overall stiffness of the series system. The maximum enhancement was limited by instability during fine-tuning near the stability boundary. This limitation was ameliorated in part by using a wider brass strip as the negative stiffness element. This allowed operation of the buckled strip with less curvature, a shallower ‘S’ shape, which gave rise to a more linear response.



**Figure 5:** (a) Negative stiffness curves for various chord lengths, L. Width of the brass strip is,  $w = 15\text{mm}$  and thickness,  $t = 0.4\text{mm}$ . Solid circles, solid triangles, solid diamonds and solid inverted triangles correspond to  $1.4 \times 10^{-2}$ ,  $2.8 \times 10^{-2}$ ,  $3.3 \times 10^{-2}$  and  $4.5 \times 10^{-2} \text{ N/mm}$  respectively. (b) Negative stiffness dependence on the change in chord length. Width of the brass strip is  $w = 15\text{mm}$  and thickness  $t = 0.4\text{mm}$ .



**Figure 6:** Sensitivity enhancement vs. the stiffness reduction. Two different widths of buckled brass strip used. Triangle corresponds to  $w = 15\text{mm}$  and circle corresponds to  $w = 30\text{mm}$ . Piezoelectric strip bend stiffness is  $4.8 \times 10^{-2} \text{ N/mm}$ .

#### 4. Conclusion

The effective piezoelectric sensitivity, displacement per volt input, increased as the inverse of the overall stiffness of the series device incorporating a negative stiffness element. The maximum observed enhancement in effective piezoelectric sensitivity was at least a factor of six, at 1 Hz.

It has been shown as a proof of concept that, by partially neutralizing the bend stiffness of the piezoelectric actuator as a cantilever using a non-piezoelectric buckled brass strip, the sensitivity response of the actuator can be enhanced. The results in this endeavor may be understood in the context of enhanced piezoelectric response by using a lattice type arrangement [15] and vibration control such as in [12] and [13] as discussed earlier.

#### Acknowledgement

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