EMA 630 VISCOELASTIC SOLIDS

Final quiz

Solve ***five*** problems and **say which five**; 20 points each. Show **logic** and state all principles and assumptions used. **Enjoy.**

Given: $\mathcal{L}[f(t)] \equiv F(s) \equiv \int_0^\infty f(t)e^{-st}dt$, $\nu = \frac{3B-2G}{6B+2G} = \frac{1}{2} - \frac{E}{6B}$, $E = 2G[1+\nu]$, $x(t) \approx x_0\, e^{-(\omega t/2)\tan\delta} \sin \omega t$. $\mathcal{L}[\frac{df(t)}{dt}] = s\mathcal{L}[f(t)] - f(0)$, $E = 2G(1+\nu)$

$\sigma(t) = \int_0^t E(t-\tau)\frac{d\varepsilon(\tau)}{d\tau}d\tau$; $\mathcal{L}[e^{-at}] = \frac{1}{s+a}$, $\mathcal{L}[1] = \frac{1}{s}$, $\mathcal{L}[\mathcal{H}(t)] = \frac{1}{s}$, $R = 1.98$ cal/moleK

$\mathcal{L}[t] = \frac{1}{s^2}$, $\mathcal{L}[\mathcal{H}(t-a)] = e^{-as}/s$, $\mathcal{L}[\delta(t-a)] = e^{-as}$, $\mathcal{L}[t^n e^{-at}] = \frac{n!}{(s+a)^{n+1}}$, $\mathcal{L}[\frac{t^{n-1}e^{at}}{(n-1)!}] = \frac{1}{(s-a)^n}$, $f(\omega) = A\frac{\omega\tau}{1 + \omega^2\tau^2}$.

$\mathcal{L}[\int_0^t f(t-\xi)g(\xi)\,d\xi] = \mathcal{L}[f(t)]\,\mathcal{L}[g(t)]$, $\mathcal{L}[\sin(at)e^{-bt}] = \frac{a}{[(s+b)^2 + a^2]}$, $\mathcal{L}[\frac{be^{bt} - ae^{at}}{(b-a)}] = \frac{s}{(s-a)(s-b)}$ for $a \neq b$, $\ln\frac{\nu_2}{\nu_1} = \frac{U}{R}\{\frac{1}{T_1} - \frac{1}{T_2}\}$.

1 Define the following using one or two sentences, or if appropriate, by an equation

(a) Stress relaxation
(b) Recovery
(c) Debye peak
(d) Resonant peak
(e) α peak
(f) Correspondence principle
(g) $\tan\delta$
(h) Resonant ultrasound spectroscopy
(i) Laplace transform
(j) Relaxation strength
(k) Reuss composite
(l) Voigt composite
(m) Boltzmann superposition integral
(n) Glass transition temperature
(o) Cross link

2 Give approximate $|E^*|$ and $\tan\delta$ values at 20°C and at 10 Hz for **ten** of these. (a) quartz, (b) natural rubber, (c) polymer at its glass transition temperature, (d) heart muscle, (e) ear cartilage, (f) steel, (g) rock, (h) PMMA, (i) elastic material, (j) ear plug foam, (k) rubbery sample provided, (l) tuning fork provided.

3 A composite material obeys the Hashin-Strikman lower formula for shear modulus $G_L = G_2 + \frac{V_1}{\frac{1}{G_1 - G_2} + \frac{6(K_2+2G_2)V_2}{5(3K_2+4G_2)G_2}}$ in which for an *elastic* composite, with K_1, K_2, G_1 and V_1, and G_2 and V_2 the bulk modulus, shear modulus, volume fraction of phases 1 and 2.

(a) Find G_L for volume fraction $V_1 = 0.5$, $G_2 = 1$ GPa, $G_1 = 100$ GPa, $\nu_1 = \nu_2 = 0.3$. Recall $K = \{2G(1+\nu)\} / \{3(1-2\nu)\}$

(b) Discuss the practical implications of this degree and kind of reinforcement for at least one such composite.

(c) Determine analytically the dynamic viscoelastic response of a *viscoelastic* composite with the same microstructure (Fig. 1).

4 For a linearly elastic tube of known outer radius R_0, inner radius R_i, length L and shear modulus G, the twisting moment M is related to the twist angle ϕ by $M = \pi(G\phi/2L)(R_0^4 - R_i^4)$.

(a) If the moment M is known as a function of time, M(t) determine the twist angle $\phi(t)$ as a function of time, for a linearly viscoelastic tube of identical dimensions. Assume any needed viscoelastic properties are known. Recall that for an elastic material, the shear compliance is $J_G = 1/G$.

(b) Determine the response to a sinusoidal load, for a linearly viscoelastic tube of identical dimensions, below resonance.

5 (a) In the frame of Figure 2, plot a Debye peak with a maximum of 0.62 and a time constant of 0.15 seconds.

(b) Explain the similarities and differences between the Debye peak and the observed properties.

(c) Discuss two applications of viscoelastic materials in which the viscoelasticity of the material is beneficial in the design. In each case, state which material properties are most important. What experimental methods over what time / frequency range are most applicable.

6 Some experimental results for a polymer foam are shown in Fig. 2. Answer the following.

(a) Sketch an instrument by which such results can be obtained and identify components. Discuss.

(b) What causes the peak in damping? Write at least one governing equation.

(c) What is the relaxation strength? Make a simplifying assumption if needed; explain what it is.

(d) What could such a material be used for? How could it alleviate suffering?

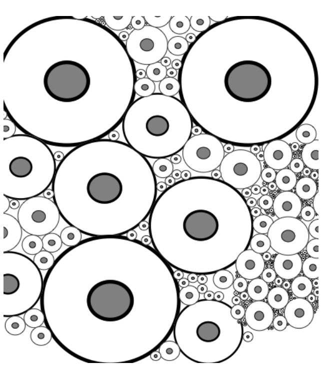

Fig. 1

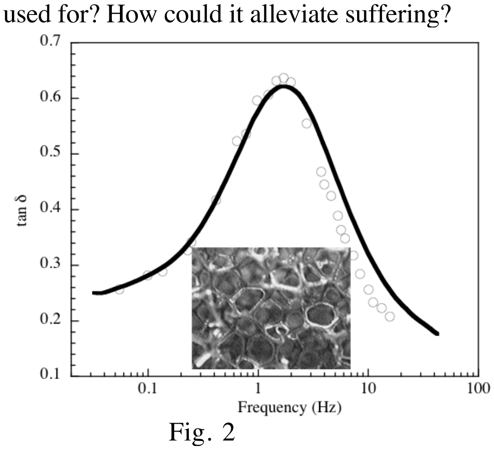

Fig. 2