VISCOELASTIC SOLIDS
Quiz 1

Show **logic** and state all principles and assumptions used.

Given: $J''(\omega) = \frac{2\omega}{\pi}\left[\wp\int_0^\infty \frac{J'(\varpi)-J'(\infty)}{\varpi^2-\omega^2}d\varpi\right]$ $\quad J'(\omega) - J'(\infty) = \frac{2}{\pi}\left[\wp\int_0^\infty \frac{\varpi J''(\varpi)}{\varpi^2-\omega^2}d\varpi\right]$, $\sigma(t) = \int_0^t E(t-\tau)\frac{d\varepsilon(\tau)}{d\tau}d\tau$; $1\text{ N} = 10^5$ dyne

$\mathcal{L}[f(t)] \equiv F(s) \equiv \int_0^\infty f(t)e^{-st}dt$, $\nu = \frac{3B-2G}{6B+2G} = \frac{1}{2} - \frac{E}{6B}$, $E = 2G[1+\nu]$, $x(t) \approx x_0\, e^{-(\omega t/2)\tan\delta}\sin\omega t$. $\mathcal{L}\left[\frac{df(t)}{dt}\right] = s\mathcal{L}[f(t)] - f(0)$, $E = 2G(1+\nu)$

$\int \frac{x}{(x^2-a^2)^2}dx = -\frac{1}{2(x^2-a^2)}$. $\text{ctn}\, x \cong \text{ctn}\,[\pi/2] + [-\csc^2(\pi/2)][x-\pi/2] + \ldots$ $\quad \mathcal{L}[e^{-at}] = \frac{1}{s+a}$, $\mathcal{L}[1] = \frac{1}{s}$, $\mathcal{L}[\boldsymbol{H}(t)] = \frac{1}{s}$,

$\mathcal{L}[t] = \frac{1}{s^2}$, $\mathcal{L}[\boldsymbol{H}(t-a)] = e^{-as}/s$, $\mathcal{L}[\delta(t-a)] = e^{-as}$, $\mathcal{L}[t^n e^{-at}] = \frac{n!}{(s+a)^{n+1}}$, $\mathcal{L}\left[\frac{t^{n-1}e^{at}}{(n-1)!}\right] = \frac{1}{(s-a)^n}$, $R = 1.98$ cal/moleK

$\mathcal{L}\left[\int_0^t f(t-\xi)g(\xi)\,d\xi\right] = \mathcal{L}[f(t)]\,\mathcal{L}[g(t)]$, $\mathcal{L}[\sin(at)e^{-bt}] = \frac{a}{[(s+b)^2+a^2]}$, $\mathcal{L}\left[\frac{be^{bt}-ae^{at}}{(b-a)}\right] = \frac{s}{(s-a)(s-b)}$ for $a \neq b$, $\ln\frac{\nu_2}{\nu_1} = \frac{U}{R}\left\{\frac{1}{T_1} - \frac{1}{T_2}\right\}$.

1 (30 pts) Define the following using one or two sentences, or if appropriate, by an equation

- (a) Creep
- (b) Stress relaxation
- (c) Recovery
- (d) Power law
- (e) Storage modulus
- (f) Time temerature superposition
- (g) aging
- (h) passive material
- (i) Dirac delta
- (j) Convolution

2 (30 pts) Show that for a linearly viscoelastic material, $\int_0^t J(t-\tau)G(\tau)\,d\tau = t$ in which G(t) is the relaxation modulus for shear and J(t) is the corresponding creep compliance.

(5 pts) Can you think of an explicit interrelation for a particular J(t)?

3 (a) (30 pts) Use the Boltzmann superposition principle to obtain [as an equation] the strain response to the following stress history. Sketch the strain history. Assume any needed mechanical properties are known.

(b) (5 pts) Develop an approximation for time t >> T for the viscoelastic response based on derivatives of the appropriate viscoelastic function.