Experimental tests of rotation sensitivity in Cosserat elasticity and in gravitation

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Preprint, R. S. Lakes, Experimental tests of rotation sensitivity in Cosserat elasticity and in gravitation, Zeitschrift fuer Angewandte Mathematik und Physik (ZAMP) 72 131 (2021).

Abstract

Experimental tests of rotation sensitivity in elastic materials and in empty space are surveyed. Pioneers in the theory for each field of study were aware of the other field. Sensitivity to rotation has been demonstrated experimentally in elastic solids but tests for rotation gradient sensitivity in gravity have found no such effects. Insight from comparison of experimental approaches may lead to new experimental protocols.

1 Introduction

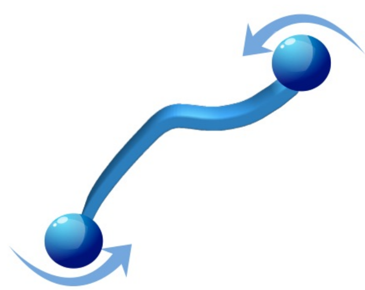

Figure 1: Points (represented by spheres) that rotate. Interaction between rotating points is represented by a curved stalk.

General relativity [1, 2] incorporates curvature of space-time but not rotation. Classical elasticity incorporates displacement of points but not their rotation. Rotation in gravitation theory was introduced by Cartan [3], who, as acknowledged, was inspired by the study of rotational freedom in elasticity theory by the Cosserat [4] brothers. Rotation dependence in general relativity was considered [5] in the context of spin; an analogy was drawn between the metric tensor of space and the strain tensor in an elastic medium. A further analogy was drawn between the couple stress (moment per area) in a Cosserat elastic solid and the spin angular momentum and between the force stress in an elastic solid and the four dimensional energy momentum in relativistic space time.

Implications of rotational freedom in gravity are reviewed in [6, 7]. We consider here the connections between rotation sensitivity (Figure 1) in Cosserat elasticity and in gravitation with focus on experiments. While many materials have been found to behave as Cosserat solids, experimental searches for rotation effects in gravity have found no such effects.

2 Gravitation

2.1 General relativity

In general relativity, mass causes gravitational effects by curving the space-time around it. The curvature depends on the presence and motion of matter and energy in the vicinity but not on spin. Briefly, the curvature is conceptualized as follows. The metric tensor relates the length or interval ds of a line element with increments dx in the coordinates. In three dimensional space, $ds^2 = g_{ij} dx_i dx_j$ in which the metric is g_{ij}. Latin subscripts refer to coordinates in three dimensions; the Einstein summation convention used for repeated indices. For Euclidean 3-D space the metric is $g_{ij} = \delta_{ij}$. In general relativity, gravitation results from curvature of four dimensional space-time [8] considered as a Riemann manifold. In the following, subscripts are covariant indices and superscripts are contravariant indices. The metric is written $g_{\mu\nu}$ with Greek subscripts or superscripts referring to the four dimensions of space-time:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\mu. \tag{1}$$

The metric in flat space-time is diagonal, $g_{00} = -1$, $g_{11} = 1$, $g_{22} = 1$, $g_{33} = 1$, $g_{44} = 1$, with the $-$ sign for time associated with the distinction between space and time in relativity.

The Einstein tensor $G_{\mu\nu}$ which governs observable phenomena in curved space-time [8] is expressed in terms of the Ricci tensor $R_{\mu\nu}$ which in turn depends on the connection Γ. The connection is expressed [8] in terms of the metric via

$$\Gamma_{\mu\beta\gamma} = \frac{1}{2}(g_{\mu\beta,\gamma} + g_{\mu\gamma,\beta} - g_{\beta\gamma,\mu}). \tag{2}$$

assuming a (holonomic) coordinate basis. The comma convention for differentiation is used. The Einstein tensor may also be viewed as a contraction of the fourth rank Riemann curvature tensor for which symmetry considerations reduce the 4^4=256 elements to 20.

In general relativity, the connection is symmetric with respect to the last two indices ($\Gamma_{\mu\beta\gamma} = \Gamma_{\mu\gamma\beta}$) in a coordinate basis because the metric is symmetric and the commutation coefficients of the holonomic coordinate basis are antisymmetric [8].

The Einstein tensor is $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ with $R = R^\alpha_\alpha$ and

$$R_{\mu\nu} = \Gamma^\alpha_{\mu\nu,\alpha} - \Gamma^\alpha_{\mu\alpha,\nu} + \Gamma^\alpha_{\beta\alpha}\Gamma^\beta_{\mu\nu} - \Gamma^\alpha_{\beta\nu}\Gamma^\beta_{\mu\alpha}. \tag{3}$$

The Einstein tensor is symmetric.

The law of gravitation is expressed in terms of the Einstein tensor $G_{\mu\nu} = kT_{\mu\nu}$ in which $T_{\mu\nu}$ is the energy momentum tensor [8] and k is a constant. In empty space,

$$G_{\mu\nu} = 0. \tag{4}$$

The gravitation law for empty space suffices for calculation of planetary orbits including the precession of the orbit of the planet Mercury. Objects moving in the vicinity of gravitating masses follow geodesics in the curved space-time governed by the Einstein equation 4.

Classic experiments to test general relativity include explanation of the observed precession of the orbit of the planet Mercury, measurement of the deflection of star light near the Sun and measurement of the gravitational red shift of electromagnetic waves. Also, gyroscopes are predicted to precess in the vicinity of a rotating mass. The effect in the vicinity of the Earth is small but has been measured with great precision using cryogenic gyroscopes in satellites [11]. Specifically, the geodetic precession rate was -6601.8 $\pm$18.3 milli arc seconds per year and the precession rate due to frame dragging was -37.2 $\pm$7.2 milli arc seconds per year. One milli arc second is about 4.85 $\times 10^{-9}$ radian. The former component of precession is due to the motion of the gyroscope with respect to the Earth and the latter is due to the effect of the moving matter in the rotating Earth [8]. The predictions of general relativity, which in this case entail an off diagonal element to the metric, were confirmed by these precession experiments. Recently gravitational waves were observed [9, 10] from distant astrophysical events, consistent with the predictions of general relativity. General relativity has been well verified experimentally in a variety of settings.

2.2 Rotation in gravitation

In the Einstein-Cartan theory, torsion of space time can occur in addition to the curvature of general relativity. Torsion is defined [12] as

$$Q^{\mu}_{\alpha\nu} = \Gamma^{\mu}_{\alpha\nu} - \Gamma^{\mu}_{\nu\alpha} \tag{5}$$

in which the connection coefficients Γ in Equation 2 are allowed to be asymmetric. Some authors [5] [7] define torsion that is half this value. Torsion is associated with rotation because [5] an orthonormal tetrad of axes at a point will rotate if it is moved in the presence of torsion. The connection relates to parallel transport of vectors. If a vector A^{ν} is parallel transported [7] along an infinitesimal increment dx^{μ}, the change due to this transport is $-A^{\nu}dx^{\mu}\Gamma^{\sigma}_{\mu\nu}$. If there is torsion, an attempt to form a parallelogram by parallel transport results in a figure which does not close. The non-closure of the parallelogram is proportional to the torsion [7]. The torsion tensor contains 24 independent components.

Torsion is hypothesized to be related to spin density $s^{\rho}_{\mu\nu}$ [13] via

$$Q^{\rho}_{\mu\nu} = 8\pi(s^{\rho}_{\mu\nu} + \frac{1}{2}\delta^{\rho}_{\mu}s^{\sigma}_{\nu\sigma} + \frac{1}{2}\delta^{\rho}_{\nu}s^{\sigma}_{\sigma\mu}) \tag{6}$$

There is considered to be no torsion of space time outside the spinning matter distribution itself [6] in the Einstein-Cartan theory. In this theory, because the torsion equation is algebraic rather than a differential equation, there is no torsion of space time outside the spinning matter distribution; the effect of torsion does not propagate. Spin, expressed as a tensor with one term [6], is considered to be intrinsic to particles; not rotation of macroscopic objects including planets. Other variants of torsion gravity allow torsion to propagate through vacuum [7]. For example, a propagating torsion theory was developed [12] and was evaluated. This theory contains a direct coupling between torsion and electromagnetic field. Further theories have been developed with different assumptions about coupling of matter with torsion.

Extensions to general relativity have also been considered in the context of Finsler [14] manifolds. Finsler manifolds are more general than the Riemann manifolds used in general relativity. The interval ds is allowed terms that can depend on direction as well as on the quadratic terms in the Riemann interval expression in Equation 1. Finsler manifolds allow torsion and they have been considered in the context of relativity [15] [16]. Finsler manifolds have been used in efforts to model dark energy in cosmology [16].

Experimental tests of extended theories of gravitation are summarized as follows.

2.3 Experimental search for rotation effects

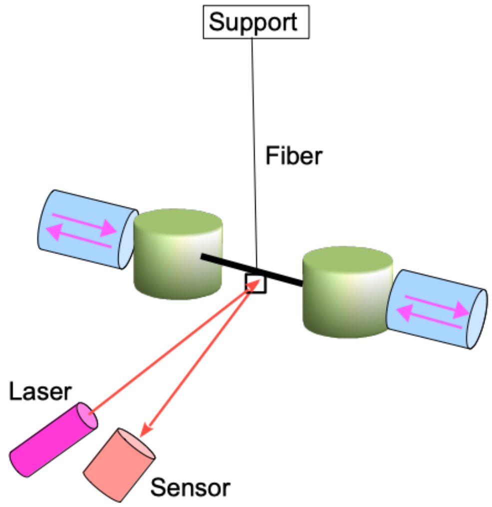

Figure 2: Torsion balance device to detect hypothetical anomalous spin dependent interaction from polarized masses (outer horizontal cylinders with arrows).

Theory [12] that predicts coupling between gravitational torsion and electromagnetism was found to be in disagreement with experiments [17]. Specifically, gravitational accelerations of materials of different composition (aluminum vs. gold or platinum) were found to be identical [18] [19] with a precision of about 3×10^{-11} as determined via highly sensitive torsion balance devices. Such balances are based on the original Cavendish balance used to determine the Newtonian gravitational constant. Torsion in a torsion balance or pendulum refers to twisting a fiber that supports two masses connected by a stalk in a dumbbell configuration; the word torsion in that context has nothing to do with torsion of space-time. Torsion balance experiments are sensitive because the large gravitational force of the Earth on the masses is decoupled. The theory predicted a difference in acceleration of the masses of dissimilar materials within the gravitational fields of the Sun and the Earth but no such effect was observed. Composition dependent tests probe hypothetical coupling between gravitation and electromagnetism because materials of different composition have a different content of electromagnetic energy. Tests for composition-dependent forces were actually done much earlier by Eötvös [20] who compared the angles of hanging plumb bobs of different materials. The direction angle depends on two components of acceleration: a downward component from the Earth's gravity and the centripetal component from the Earth's rotation. Further experiments [21] improved the precision; precision was within 3×10^{-9}. Prior to that, experiments of Galileo disclosed no effect of composition on gravitational acceleration of objects. In all such tests, no composition dependent force was found.

The search for anomalous spin interactions is challenging in view of the fact that known magnetic interactions between spins of electrons or nuclei exceed such anomalous interactions by at least a factor of 10^{11} [22]. Use of Dy_6Fe_{23} enabled the experimenter to subtract magnetic effects due to spin from magnetic effects due to orbital motion of electrons in the material. Combined with magnetic shielding this approach enabled a rigorous limit to be set upon interactions between spins. Further experimental upper limits were set on anomalous spin dependent forces [23] using a torsion pendulum with spin-polarized Dy_6Fe_{23} and copper masses. Such experiments make use of a variant of the classic Cavendish balance approach as illustrated in Figure 2. In the similar device used in

[18], an effect of gravitation from the Sun was sought so there were no outer horizontal cylinders of the sort shown in the diagram. Limits on electron spin interactions and on moments which are not of electromagnetic origin were set [24] by probing interaction of spins in a spin-polarized test mass and those in a paramagnetic salt. Objects were separated using a mu-metal magnetic shield and a superconducting shield. The limit was less than 3×10^{-14} of the interaction between the magnetic moments of the electrons.

Studies of gravitational torsion have been reviewed in [7]. There are many theories and formulations of torsion gravity and in the assumed interaction between spins, including many theories in which torsion propagates through vacuum. The proliferation of theories is, however, considered to be a challenge in seeking experimental evidence.

It has generally been believed that torsion can couple only to the intrinsic spin of elementary particles, not to rotational angular momentum. It was argued that this assumption has a logical loophole [25] such that study of effects of rotating bodies in the solar system on gyroscopes is warranted. Limits are placed on gravitational torsion effects predicted by several theories, from observations within the solar system. Moreover, measurements of the Moon?s geodetic precession, and the measurements of Mercury?s perihelion advance were used to constrain combinations of torsion parameters [26].

Recent experimental searches for Lorentz violation were exploited to extract new constraints involving 19 of the 24 independent torsion tensor components and upper bounds have been placed down to energy levels on the order 10^{-31} GeV [27]. Nonzero torsion over a region of space-time entails a preferred orientation for a freely falling object. This is the criterion for Lorentz violation.

Further limits [28] were placed upon sensitivity to hypothetical dipole-dipole, spin-dot-spin, and spin-cross-spin exchange interactions. This was done with a rotating torsion pendulum with samarium cobalt and Alnico permanent magnets, magnetized to the same degree, so that the magnetic fields contained nominally closed loops without external flux.

In summary, experimental searches for rotation effects in gravity and for anomalous interactions between spins have found no such effects.

3 Electromagnetism models

Classical electromagnetism is presently conceptualized using Maxwell's equations. In the 19th century, pioneers in electromagnetic theory, including Maxwell, considered it helpful to formulate mechanical models to help visualize the phenomena. Moreover, such models were used to envisage the medium, called aether, in which electromagnetic phenomena occur. These models are of interest in the context of present day mechanics as well as materials science because they incorporate extreme elastic behavior and elastic behavior that is not subsumed within classical elasticity.

3.1 Compressibility

Sound waves propagate in media such as solids, liquids and gases but they do not propagate through vacuum. Electromagnetic waves such as light and radio waves do propagate through vacuum as well as through some solids, liquids and gases. It was once considered necessary that light have a medium called aether in which to propagate. Elastic solids were initially considered as models for the aether [29] but that was recognized as problematical because light waves are transverse and elastic solids admit both longitudinal and transverse waves. To eliminate the longitudinal waves, one can force their velocity to zero or infinity. Kelvin [30] envisaged an elastic medium with a negative compressibility so that longitudinal waves could not propagate. Such a medium is unstable. An airless foam restrained from collapse by adhesion to a container was proposed as a physical embodiment. Polymer foams were unknown at that time so the reference was likely to a soap foam. A compressibility that diverges (zero bulk modulus) does not suffice to eliminate longitudinal waves for reasons discussed below in the context of elasticity. If, by contrast, the compressibility tends to zero (bulk modulus diverges; an incompressible medium), then longitudinal wave velocity diverges. Kelvin [31] envisaged a lattice containing spheres at the ends of ribs or bars, four ribs per sphere; the spheres were free to rotate on smooth pivots. This lattice embodies an incompressible medium but there is no resistance to shear so there are no transverse waves.

3.2 Rotation

To provide for transverse waves, Kelvin [31], moreover, considered adding gyroscopes to the above structure of ribs and spheres in order to contribute rigidity with respect to rotation of nodes but not translation. Rotation sensitivity was also considered by Maxwell [32] in mechanical models for visualizing the magnetic field. A related model of FitzGerald contained pivoted circular rotors linked by rubber bands [33]. In this model, rotation of one rotor causes all of them to rotate in the same direction unless the bands slip. A physical model was constructed and used for lecture demonstrations [34].

MacCullagh [35] introduced an elastic solid in which the potential energy depends only on rotation of volume elements, not translation. This medium admits only transverse waves; behavior of the waves agrees with observations of electromagnetic waves such as light, including reflection and refraction at interfaces. The theory is a continuum theory and does not incorporate a mechanical model.

Elastic models for the aether do not account for the fact that solid objects move through the vacuum without resistance. Also, the aether does not behave as a gas or a liquid: the Michelson-Morley experiment and related experiments showed that the Earth's motion cannot be detected via laboratory experiments. Specifically, the speed of light is independent of the motion of the source or of the detector as incorporated in special relativity. The notion that physics occurs in vacuum persists in our modern view: we have general relativity and quantum mechanics. One currently speaks of relativistic space-time and the quantum vacuum rather than aether.

The MacCullagh [35] aether continuum model is of interest because it corresponds to an extreme case of Cosserat elasticity in which the rotation sensitivity greatly exceeds the sensitivity to strain. Further details are presented in the following section.

4 Elasticity

In this section, the freedom associated with elastic deformation is explored, with focus on the relations between theory and experiment, and comparison with corresponding efforts in gravitation.

4.1 Classical elasticity

In classical elasticity the displacement of points is considered. The symmetric part of the gradient of the displacement u gives the strain $\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$. Stress σ_{ij} is proportional to strain in isotropic linear elasticity [36] as follows.

$$\sigma_{ij} = 2G\epsilon_{ij} + \lambda\epsilon_{kk}\delta_{ij}. \tag{7}$$

G is the shear modulus and λ is a Lamé elastic constant. In classical elasticity the stress is symmetric. The bulk modulus (inverse compressibility) is given by $B = \lambda + \frac{2}{3}G$. Stability considerations have been used to impose bounds on elastic moduli: compressibility, shear modulus and Poisson's ratio. For an unconstrained material to be stable, the bulk modulus B is positive and the shear modulus G is also positive. Because the elastic constants are interrelated in an isotropic material, the bounds on Poisson's ratio ν are $-1 < \nu < 0.5$.

The limit of an incompressible material can be approached physically via a soft rubber with a low concentration of cross links. One may also approach the limit of incompressibility via lattices of ribs; a hinged rib lattice of this type [37] organized in a diamond-like structure was anticipated by Kelvin [31] in the aether model described above. If the hinges are perfect, the shear modulus is zero and the lattice is incompressible. A lattice of ribs organized as repeating truncated octahedra (tetrakaidecahedra) [38] exhibits $B >> G$ if the ribs are slender; this lattice also approaches incompressibility.

The limit of a highly compressible material $B << G$, which corresponds to a negative Poisson's ratio tending to -1, has been approached in designed polymer and metal foams [39]. These materials are stable. Structures containing hinged tilting squares [40] or tilting cubes [41] can also attain extreme negative Poisson's ratio provided the hinges are perfect; these are neutrally stable.

Negative compressibility stabilized by constraint had been suggested earlier [30] in the context of an aether model discussed above. In the context of more recent composites, constrained materials with negative compressibility have been demonstrated experimentally in composites and shown to exhibit large viscoelastic damping [42, 43] and large stiffness [44]. Negative incremental compressibility was also demonstrated experimentally [45] in a constrained polymer foam.

4.2 Cosserat elasticity

Cosserat [4] [46] elastic solids incorporate freedom of rotation of points as well as translation of points in the material. They admit couple stress (moment per area) as well as force stress (force per area). Cosserat solids with rotational inertia are called micropolar [47]. The constitutive equations for linear isotropic Cosserat elasticity are, with e_{jkm} as the permutation symbol:

$$\sigma_{ij} = 2G\epsilon_{ij} + \lambda\epsilon_{kk}\delta_{ij} + \kappa e_{ijk}(r_k - \phi_k) \tag{8}$$

$$m_{ij} = \alpha\phi_{k,k}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}. \tag{9}$$

The points in the Cosserat continuum have rotational freedom ϕ_k, called micro-rotation. In general ϕ_k differs from $r_k = \frac{1}{2}e_{klm}u_{m,l}$, the "macro" rotation associated with the antisymmetric part of gradient of displacement u_i. The Cauchy stress σ_{ij} (force per unit area) in Cosserat elasticity can be asymmetric. Cosserat theory admits a couple stress m_{ij} (a torque per unit area) which balances the moment associated with asymmetric stress. The moments generated by a rotation gradient may be envisaged as in Figure 1.

The six Cosserat elastic constants are λ, G, α, β, γ, κ. Elastic constants λ and G have the same meaning as in classical elasticity. G is the shear modulus in the absence of gradients. Constants α, β, γ allow sensitivity to gradient of micro-rotation. Constant κ quantifies the degree of coupling between micro and macro rotation fields. If $\kappa \to \infty$, then $\phi_k \to r_k$ which corresponds to a special case of the Cosserat theory called couple stress theory [48]. In that case, the antisymmetric part of the stress σ_{ij} and the invariant m_{kk} become indeterminate.

Mathematical models of a Cosserat solid [49, 50] in 2D and 3D were envisaged with rotatable nodes connected by rubber bands at the periphery and by springs at the centers. The 2D model is similar to an aether model containing circular rotors linked by rubber bands via FitzGerald [33], that was originally intended to model the magnetic field. The MacCullagh [35] aether introduced in 1848 as a continuum elastic model for electromagnetism corresponds to an extreme Cosserat elastic solid in which sensitivity to rotation gradients greatly exceeds the usual elastic response. Any

Cosserat rotation effect occurs through a solid medium. One has freedom to design Cosserat solids but there is no such freedom with empty space, the relativistic vacuum. Cosserat rotation is an independent kinematical variable, a vector with three components in three dimensions independent of the rotation associated with motion of neighboring points. By contrast, in torsion gravity, there is one rotation associated with deformation of space-time represented by the torsion tensor which has 24 independent components.

Theoretical links between Cartan torsion of space-time and Cosserat elasticity were explored [51]. In the Cosserat continuum, the rotation associated with motion of neighboring points was considered and links with the theory of dislocations in crystals were explored.

The earlier experimental studies of Cosserat elasticity entailed choice of available materials. One may surmise that a composite with nodules that can rotate within a more compliant matrix would exhibit Cosserat elastic effects. However a composite of aluminum beads in an epoxy polymer matrix was found experimentally to obey classical elasticity not Cosserat elasticity [52]. Indeed, analysis of composites containing stiff spheres in a more compliant matrix revealed classical behavior [53], consistent with these experiments. In early experiments, Cosserat effects were sought in polycrystalline metals such as aluminum and steel, but no Cosserat effects were found; the metals obeyed classical elasticity [54, 55]. Compact bone was studied in view of its comparatively large fibers (0.2-0.3 mm diameter) with a weak and compliant interface between them. Bone was found to exhibit Cosserat elastic effects [56]. A dense polymer foam was found to exhibit Cosserat elastic effects [57, 58]; all six elastic constants were extracted from experimental results. Rotational waves of the kind predicted in Cosserat elasticity were observed in a non-cohesive packing of metal spheres [59].

As for planning and interpreting experiments, Cosserat effects are predicted to be manifested in nonclassical size dependence of rigidity in torsion and bending [52]. In classical elasticity the structural rigidity is proportional to d^4 with d as the diameter or width; in the simplest form ($\kappa \to \infty$), Cosserat elasticity contributes a d^2 term to the rigidity from distributed moments. Asymmetry of the stress can be inferred from behavior near boundaries of Cosserat elastic solids. Rotational waves can occur in Cosserat solids [47]; also dispersive shear waves. However interpretation of wave experiments can be challenging if the material is also viscoelastic because viscoelasticity also gives rise to dispersion of waves. Exterior corners of square section bars in torsion must have zero stress and zero strain in classical elasticity because there are two orthogonal free surfaces that meet at an edge. Nonzero stress or strain inferred at the corners in experiments is nonclassical and reveals Cosserat elasticity [60, 61, 62]. Materials, including a variety of rib lattices and surface lattices, have been developed to exhibit strong Cosserat effects. Forces (straight arrows) and moments (curved arrows) can be visualized acting on ribs of a lattice as illustrated in Figure 3. In the Cosserat continuum view, the average force per area is the stress and the average moment per area is the couple stress. A polymer lattice containing shaped ribs [63] exhibited deviations of about a factor 30 with respect to predictions of classical elasticity; this exceeds the factor of 6.5 observed in a foam [64] with low density. A designed anisotropic fibrous material with shaped fibers [65] exhibited larger deviations of about a factor 128 compared with classical predictions. A material comprised of tilting hinged cubes [41] exhibits extremal negative Poisson's ratio and extremal Cosserat elasticity. Chiral lattices [66] [67] exhibit nonclassical effects (coupling between compression and rotation) that cannot be subsumed within classical elasticity but are understood within Cosserat elasticity.

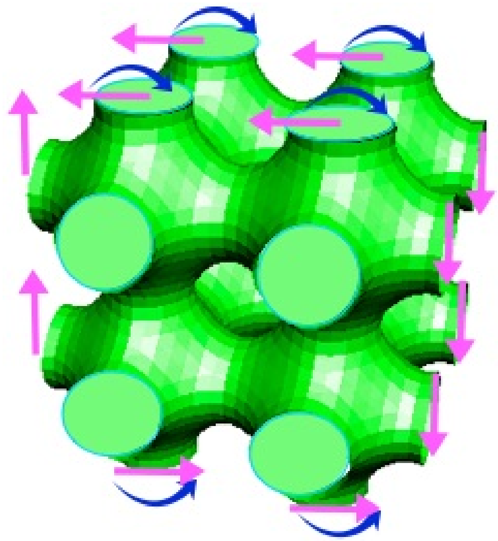

Figure 3: Conceptual view of a Cosserat solid as a lattice, supporting forces and moments.

4.3 Comparisons, elasticity and gravitation

Cosserat elasticity has two independent rotation variables, the rotation of points themselves and the rotation associated with motion of nearby points. There are 6 isotropic elastic constants which have been determined experimentally for some materials. By contrast, torsion gravity rotation is based on the rotation of a tetrad of axes displaced through space-time. Although there is only one kind of rotation, the torsion tensor has 24 components. Some of these have been bounded via experimental results.

In both classical elasticity and Cosserat elasticity, the source of deformation is either generalized traction or displacement applied on the surface. Effects then propagate through the material. By contrast, an analogy was drawn between the metric tensor of space-time and the strain tensor in an elastic medium [5] and the stress in an elastic solid and the four dimensional energy-momentum in relativistic space-time. However the stress and strain are proportional in elastic solids. The source in elasticity is actually the imposed traction or displacement at the surface, not the stress at an arbitrary point in the elastic material. The source term in general relativity is the energy-momentum tensor which includes gravitating mass.

Elasticity theories always apply to solid materials. One may choose from available materials or one may design new materials; in either case, effects which may or may not be classical will propagate through the material. In the search for anomalous gravitational effects or related interactions between spins, there is also a choice of materials, but there is no option to control the interaction through vacuum.

Cosserat elasticity allows an asymmetric stress tensor in contrast with classical elasticity in which the stress must be symmetric. Similarly in torsion gravity, the torsion is the antisymmetric part of the connection which is symmetric in general relativity. In Cosserat elasticity the asymmetry of the stress can be directly inferred. The inference requires a free surface, an interface between a solid material and either air or vacuum. There is no counterpart to such a free surface in gravitation.

Experimental methods in both gravity and in elasticity seek nonclassical effects in the presence of known classical effects. In both kinds of experiment, one needs not only a theory but also a specific theoretical prediction of phenomena. There is a proliferation of extended theories in both gravitation and elasticity; these are not always accompanied by testable predictions. In both kinds of experiment, one seeks to minimize the effect of known classical forces and to maximize the sensitivity to non-classical effects.

Experimental efforts in torsion gravity have by necessity sought to observe exceptionally small effects. It is possible to pursue a similar path in Cosserat elasticity but experiments to date have focused on choice of materials or design of materials that exhibit large effects. Because there is a characteristic length scale in Cosserat elasticity, materials expected to have a characteristic length on the nano-scale can be observed on that scale so the effects are easily observable. In principle one could perform a macroscopic experiment to reveal a nano-scale characteristic length by observing stretch-twist coupling in a chiral fiber using a sensitive torsion balance or by closely measuring strain near a corner where classically the strain should be exactly zero. Conversely, a torsion gravity theory with two independent rotations may be envisaged by analogy to Cosserat elasticity. While in Cosserat elasticity there is physical insight in support of two rotation variables, that is not so obvious in the case of gravitation.

5 Conclusion

Rotation variables are incorporated in both Cosserat elasticity and in torsion gravity. The pioneers in the analytical aspects of each field of study were aware of the other field. Early efforts to understand the physics of electromagnetic fields let to the development of mechanical models that have counterparts in materials science today. Experiments to reveal rotation sensitivity in both fields of study have some similarities. One seeks to minimize the magnitude of the known classical effects in comparison with sensitivity to nonclassical effects. Experiments have disclosed Cosserat elastic effects in a variety of solids; in some cases, strong effects. No evidence of torsion gravity has been found to date. One may seek to envisage new experimental protocols in view of the comparisons between Cosserat elasticity and torsion gravity.

The authors gratefully acknowledge support of this research by the National Science Foundation via Grant No. CMMI -1906890.

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