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## Transient Study of Couple Stress Effects in Compact Bone: Torsion

*Couple stress theory, which admits an internal moment per unit area as well as the usual force per unit area, is a generalization of classical elasticity. Experimentally we have demonstrated the existence of couple stress by measuring the effect of size on apparent stiffness of compact bone in quasi-static torsion. From these measurements, we obtain the characteristic length for bone in couple stress theory.*

### Introduction

In classical continuum theory, the basic assumptions are that the medium is continuous and the constitutive equations have to meet the principle of determination, local action and objectivity. The assumption of continuous media means the atomic, molecular, pore and grain structure of real materials is to be disregarded. Under such assumptions, all material properties can be defined as a continuous point function. The classical continuum theory of elasticity has been very successful in its determination of stress and strain fields in most engineering materials. But it fails to correctly predict the fatigue behavior of machine elements with high stress gradient [1]. The discrepancy between prediction and observation is more marked for materials with a coarse grain structure [2]. A possible explanation is that the constitutive equation of classical elasticity theory may not be sufficiently general. To remedy this, several generalized continuum theories have been introduced which have additional degrees of freedom. One such theory is the couple stress theory introduced by the Cosserats [3], and developed further by Mindlin, Tiersten [4] and others.

The basic distinction between the classical elasticity theory and the couple stress theory is the nature of the assumed interaction of the material on the surface element. The assumption, neither proved nor disproved, in classical theory, is that the load acting on the infinitesimal surface element is described only by a force vector. In the couple stress theory, one assumes that the surface element may transmit both a force vector and a couple vector. The stress, which in this theory may become asymmetric, depends not only on the strain but on some elements of the strain gradient tensor [5]. Some important problems have been solved by several authors under this theory, and it is found that the couple stress effects become significant when some physical dimension of the body approaches a certain characteristic length which is an additional material coefficient. Mindlin [6] has investigated the effect of couple stress in the stress concentration problem. He

found that stress concentration factors for holes calculated from couple stress theory are less than the factors obtained from classical elasticity. The reduction in the stress concentration factor is significant when the hole radius approaches the characteristic length. Micropolar theory, introduced by Eringen [7, 8], also admits an asymmetric stress tensor, and includes degrees of freedom not present in the couple stress theory or classical theory of elasticity. As in couple stress theory, the rigidity of slender rods in bending [9] and in torsion [10] is predicted to be greater than what is expected on the basis of classical elasticity.

Theoretical investigations have dealt not only with the solution of boundary value problems in extended continuum theories, but also with the calculation of the material coefficients in such theories from structural considerations. Continuum models have been developed for laminates [11], lattice structures built up from elastic beams [12], and a structure composed of elastic cubical cells [13]. In such models, the characteristic length  $l$  is related to the size of the structural elements. Bone, a natural fibrous composite, has also been modeled in light of extended continuum theory. It has been suggested that cancellous [14] and compact [15] bone may obey couple stress theory.

Experimental work supporting couple stress and related theories has been sparse. The classical theory of elasticity has been most successful in dealing with the usual engineering materials below the yield point. Experiments seeking to measure couple stress effects in aluminum [16, 17], steel [17], and a model composite [10] have yielded results consistent with the classical theory. Optical studies on crystalline  $\text{KNO}_3$ , interpreted in light of micropolar theory [18] suggest a characteristic length of the order of the lattice parameter, far too small to be observable in a macroscopic mechanical situation. Study of a foam material [19] suggests that couple stress effects may occur. The method used loaded resonance of a hollow cylindrical layer of foam, in which layer thickness was an independent variable. This method has the drawback that time/frequency dependence (viscoelasticity) and spatial derivatives (couple stress theory) in the stress strain relation both contribute to an effective stiffening of the layer. As the

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layer thickness is reduced, both the strain gradient and the resonant frequency of the system increases. Thus, since the polyvinyl chloride foam used is strongly viscoelastic, the interpretation of the results in terms of couple stress theory is far from straightforward.

### Theory

The constitutive equations of linear, isotropic couple stress theory are [5]:

$$\sigma_{ij}^s = \lambda \epsilon_{kk} \delta_{ij} + 2\mu \epsilon_{ij} \quad (1)$$

$$\mu_{ij}^d = 4\eta \kappa_{ij} + 4\eta' \kappa_{ji}, \quad (2)$$

in which  $\sigma^s$  is the symmetric part of the force stress tensor,  $\mu^d$  is the deviator of the couple stress tensor,  $\epsilon$  is the strain,  $\kappa$  is the curvature,  $\lambda$  and  $\mu$  are Lamé constants and  $\eta$  and  $\eta'$  are additional elastic constants associated with couple stress theory. The curvature  $\kappa$  is related to the strain gradient by

$$\kappa_{ml} = e_{nkl} \epsilon_{mk,n}, \quad (2)$$

in which  $e_{nkl}$  is the alternating symbol. The quantity  $l = \sqrt{\eta/\mu}$  has dimension of length and is the characteristic length of couple stress theory. The quantity  $\eta'/\eta$  represents a ratio of curvatures [4] and is contained in the interval  $[-1, 1]$ . The theory has been applied by Koiter [1] to the case of torsion of a long circular cylinder of diameter  $d$  length  $z$ :

$$\frac{\tau}{\theta} = \frac{\pi}{32} G \frac{d^4}{z} + \frac{3\pi}{2} \left(1 + \frac{\eta'}{\eta}\right) G l^2 \frac{d^2}{z}, \quad (4)$$

in which  $\tau$  is the applied torque,  $\theta$  is the angular displacement,  $l$  is the characteristic length of couple stress theory,  $G$  is the shear modulus. Koiter adopts a modified notation in which  $\eta'/\eta$  is replaced by  $\eta$ ; here we use the same symbols written in equations (1) and (2). For the special case of classical (force stress) elasticity, the quantities  $l$  and  $\eta'$  vanish and the elementary result for torsion of an elastic rod is recovered. Based on the foregoing result, one should be able to distinguish between couple stress elastic behavior and classical elastic behavior by measuring the torsional rigidity  $\tau z/\theta$  of rods of different diameter. The material coefficients  $G$  and  $(1 + \eta'/\eta)l^2 \equiv l_t^2$  may be extracted straightforwardly from a plot of  $(\tau z/\theta) (1/d^2)$  versus  $d^2$ . If couple stress elasticity describes the material, the graph is a straight line of slope  $(\pi/32)G$  and  $y$ -intercept  $(3/2)\pi G l_t^2$ . If the material is elastic, the characteristic length in torsion  $l_t$  is zero and the straight line passes through the origin. Koiter's result may appear counterintuitive to some observers since it predicts an apparent shear modulus which increases without bound as the specimen size is reduced. In a real material, the couple stress theory would most likely break down for specimens of size equal to or smaller than the characteristic length; a more general theory would be needed.

### Instrumentation and Procedure

To test the hypothesis that bone obeys couple stress theory, cylindrical specimens of compact bone were tested in torsion, machined to a smaller diameter and tested again. The procedure was repeated until the specimen broke or became too thin to handle effectively. To separate viscoelastic effects from couple stress effects, isochronal data from the successive torsion creep experiments were used in the analysis of size effects.

Specimens of compact bone were obtained from human long bones from donated autopsy tissue. Specimens were cut slowly, while wet, on a precision lathe into a cylindrical shape with flared, threaded ends as shown in Fig. 1. The long axis of each specimen was parallel to the bone axis. Bone specimens 1-4 were from the femur of a male 175 cm (5 ft 9 in.) in height, 59.1 kg in mass (130 lb weight), who died at age 27 of

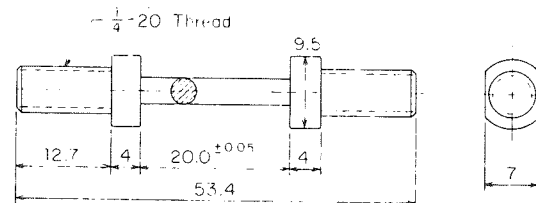


Fig. 1 Specimen configuration - dimensions are in mm

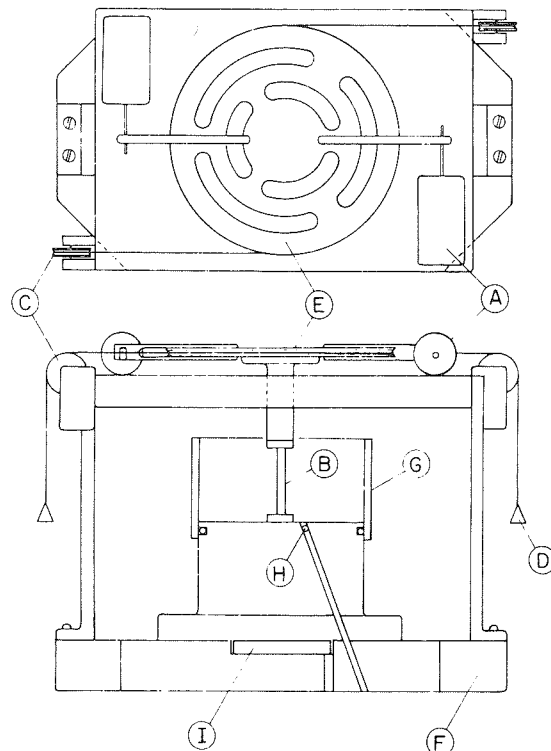


Fig. 2 Experimental apparatus - A barrel of free-core LVDT, B bone specimen, C ball-bearing pulley, D dead weight, E torque rotor, F base plate, G plexiglass cell, H temperature probe, I thermoelectric heater

hepatic failure. Specimens 5, 6 were from the right femur of a male 178 cm (5ft 10in.) in height, 72.7 kg in mass (180 lb weight) who died at age 57 of Hodgkin's disease. During the course of experiments the specimens were kept in Ringer's solution with a bactericidal additive, and maintained at body temperature ( $36.5^\circ\text{C} \pm 0.2^\circ\text{C}$ ) by means of a closed loop temperature controller of original design. Specimens of polymethylmethacrylate (PMMA) were also prepared, and were tested dry at room temperature ( $22.5^\circ\text{C}$ ).

One end of the specimen was fixed in the base plate as shown in Fig. 2. The other end was mounted in a loading wheel which enabled a step torque to be applied by means of an arrangement of dead weights and low friction, ball bearing pulleys. Step loading was applied with a risetime of less than one second and sustained for periods of time greater than 240 s. The base plate was mounted on a special foundation which was designed to isolate the instrument from external vibration and shock. To minimize friction, precision ball bearings used in the pulleys were vibrated in "dither" at 3.4 kHz by magnetic coils or by piezoelectric transducers. The magnetic vibrator was more effective.

The angle of twist  $\omega$  determined from linear displacement measurements made using two DC to DC LVDT's, the cores of which were mounted on the torque wheel in the direction tangential to the wheel and were powered by a Tektronix power module, TM506. The LVDT's were calibrated using a

**Table 1**

Specimen	Load time (s)	A(N)	B(MN/m <sup>2</sup> )	G(GN/m <sup>2</sup> )	l <sub>c</sub> (mm)	Time of test (days)
Bone no. 1	0.3	488	534	5.44	0.14	7
	60	427	513	5.23	0.13	
Bone no. 2	0.3	263	423	4.31	0.11	38
	120	156	408	4.15	0.09	
	240	165	401	4.09	0.09	
Bone no. 3	0.3	273	408	4.16	0.12	21
	120	292	373	3.80	0.13	
	240	306	364	3.71	0.13	
Bone no. 4	0.3	433	402	4.09	0.15	13
	120	366	385	3.92	0.14	
	240	362	379	3.86	0.14	
	1800	368	366	3.73	0.145	
Bone no. 5	0.3	656	344	3.50	0.20	13
	120	682	318	3.24	0.21	
	240	677	314	3.20	0.21	
	1800	552	327	3.33	0.19	
Bone no. 6	0.3	704	347	3.53	0.21	15
	120	762	309	3.15	0.23	
	240	768	303	3.09	0.23	
	1800	686	304	3.09	0.22	
PMMA	0.3	-23	124	1.26		9
	24	-27	111	1.13		
	72	-15	106	1.08		
	120	-17	105	1.07		

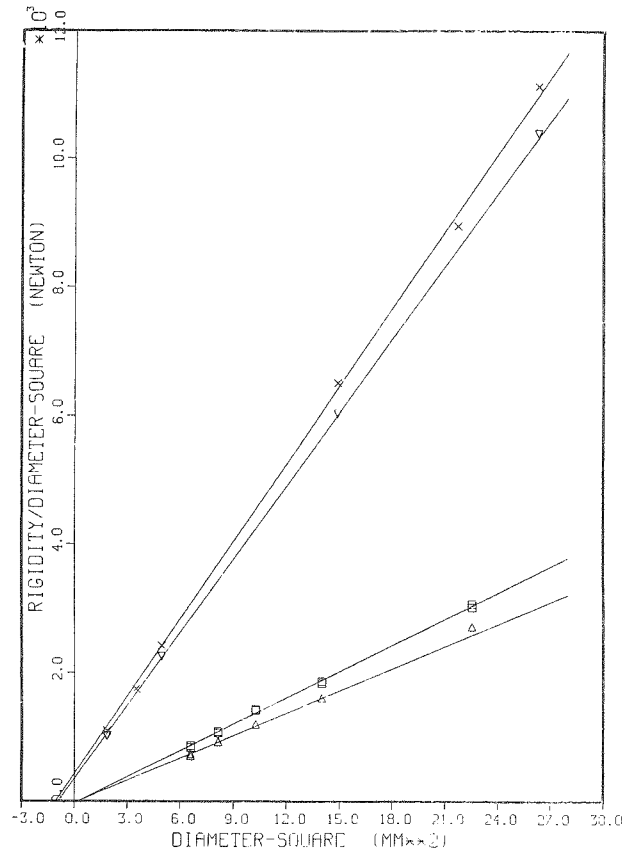
bench micrometer. The LVDT outputs were connected to a two-channel chart recorder (Gould). It was necessary to eliminate any effect of parasitic bending of the specimen since the top torque rotor was unconstrained, to minimize friction. The LVDT outputs were summed to eliminate any contribution from parasitic bending. In addition, microscopic examination of the center of the torque rotor revealed no significant bending. Linearity of response was checked by repeating the tests at different load levels. Maximum strain did not exceed  $10^{-4}$  which ensured both material and geometrical linear behavior. Sufficient time for creep recovery was allowed between tests. Specimens were also allowed to recover strains introduced in machining, after they were cut to a smaller size. Straight lines based on equation (4) were fitted to the data by defining  $y \equiv (\tau z / \theta) (1/d^2)$ ,  $x \equiv d^2$  and using a least-squares analysis to obtain the coefficients  $B$  and  $A$  in  $y = Bx + A$ .

**Results**

Results are shown in Table 1. Several typical graphs illustrating equation (4) are shown in Fig. 3. For the polymethylmethacrylate (PMMA), the isochronal curves for  $t = 0.3$  s, 24 s, 72 s, and 120 s all pass through the origin within the limits of experimental scatter. The scatter may be explained by allowing for reasonable tolerance in specimen manufacture, specimen preparation and measurement error.

Compact bone exhibits nonuniformities over a cross section; therefore measurements of mechanical properties can be expected to exhibit some additional scatter. To reduce the effects of this scatter, several specimens must be examined. In this experiment, we tested six specimens of compact bone. During testing of specimen no. 1, we did not have temperature controller, so we tested it at room temperature ( $24.4 \pm 0.5^\circ\text{C}$ ). From Table 1, we see that for all six specimens the isochronal curves corresponding to  $t = 0.3$  s, 120 s, 240 s, and 1,800 s, do not pass through the origin since the offsets  $A$  are not zero. The mean value of the characteristic length of compact bone at 0.3 s is 0.155 mm, which is of the order of the radius of an osteon.

Confidence intervals for the regression coefficients are given in Table 2. Times of loading are as in Table 1. Coefficient  $A$  is related to the characteristic length and  $B$  is related



**Fig. 3 Behavior of bone and PMMA**  
 □ PMMA,  $t \approx 0.3$  s  
 △ PMMA,  $t \approx 120$  s  
 × Bone 4,  $t = 0.3$  s  
 ▽ Bone 4,  $t = 240$  s

to the shear modulus by equation (4). For all bone specimens except 3, we can state with better than 95 percent confidence that the regression line does not pass through the origin, i.e., that the specimen obeys the couple stress theory rather than the classical theory of elasticity. For four of the specimens, this one-sided confidence limit is better than 99 percent. For

Table 2

Specimen	Confidence interval for line	Points	Confidence interval				
			Not passing origin	90 percent		95 percent	
				A	B	A	B
Bone no. 1	98%	11	488 ± 312	534 ± 26	488 ± 385	534 ± 32	
	75%	6	427 ± 680	513 ± 51	427 ± 886	513 ± 67	
Bone no. 2	99.5%	10	263 ± 125	423 ± 10	263 ± 155	423 ± 12.5	
	80%	10	156 ± 208	408 ± 16	156 ± 258	408 ± 20	
	75%	10	165 ± 238	401 ± 18	165 ± 295	401 ± 23	
Bone no. 3	78%	12	273 ± 377	408 ± 31	273 ± 463	408 ± 38	
	78%	12	292 ± 406	373 ± 33	292 ± 499	373 ± 40	
	78.5%	12	306 ± 419	364 ± 34	306 ± 515	364 ± 42	
Bone no. 4	99.9%	15	433 ± 122	402 ± 8	433 ± 149	402 ± 10	
	99.9%	14	366 ± 78	385 ± 5	366 ± 96	385 ± 7	
	99.9%	14	362 ± 77	379 ± 5	362 ± 94	379 ± 6	
	93%	5	368 ± 322	366 ± 4	368 ± 436	366 ± 54	
Bone no. 5	99.5%	9	656 ± 311	344 ± 23	656 ± 388	344 ± 28	
	99.6%	9	682 ± 290	318 ± 21	682 ± 362	318 ± 26	
	99.6%	9	677 ± 290	314 ± 21	677 ± 362	314 ± 26	
	94%	3	552 ± 341	327 ± 33	552 ± 686	327 ± 67	
Bone no. 6	99.8%	10	704 ± 266	347 ± 22	704 ± 330	347 ± 28	
	99.9%	10	762 ± 260	309 ± 20	762 ± 323	309 ± 25	
	99.9%	10	768 ± 249	303 ± 20	768 ± 309	303 ± 25	
	96%	5	686 ± 358	304 ± 28	686 ± 484	304 ± 38	
PMMA	63%	10	-23 ± 47	124 ± 3.5	-23 ± 58	124 ± 4.4	
	55%	10	-27 ± 60	111 ± 4.3	-27 ± 74	111 ± 5.3	
	25%	10	-15 ± 82	106 ± 6.1	-15 ± 101	106 ± 7.6	
	26%	10	-17 ± 87	105 ± 6.5	-17 ± 108	105 ± 8.1	

PMMA, the behavior is classical within the experimental scatter. The characteristic length of the bone specimens appears to be independent of the time following application of the load.

The testing periods for these six bone specimens are given in Table 1. The effect of soaking in Ringer's solution in dissolving mineral and softening the specimen is more severe for thin than for thick specimens. This would lead to an underestimate of the characteristic length. The soaking artifact can be reduced by the time of immersion, but the necessity for repeated tests places a lower bound on this time. Analysis of the relationship between the characteristic length for initial loading (0.3 s) and total time  $T$  of immersion during testing yields the following regression line:  $l_c(\text{mm}) = 0.191 - 0.00205 T$  (days). The soaking artifact indeed appears to be associated with a reduction in the apparent characteristic length. The regression equation suggests that if all the testing could have been done in zero days, the average value of the characteristic length would be 0.191 mm rather than 0.155 mm actually observed.

## Discussion and Conclusion

Human compact bone appears to behave in a fashion describable by couple stress theory. Polymethylmethacrylate (PMMA) obeys the classical theory of viscoelasticity. We attribute this difference in behavior to the fact that bone is a structured material, the major structural elements of which, the osteons, are ca 0.15–0.25 mm in diameter. PMMA is an amorphous polymer, the major structural elements of which are on the molecular scale.

The shear moduli determined in this study may be compared with the literature. For PMMA at a temperature near 25°C, the dynamic shear modulus in the range  $10^{-2}$  to  $10^{-1}$  Hz is about 1.1 GN/m<sup>2</sup> [20]. For wet human tibial bone at 37°C, in relaxation at 10 s,  $G$  is about 4.15 GN/m<sup>2</sup> [21]. In this approximate comparison, interchange of the viscoelastic functions is justified on the basis of the smallness of time-dependent response in bone and PMMA at these times and

temperatures. The relatively large shear modulus exhibited by specimen no. 1 in this study presumably results from the fact that it was tested at room temperature rather than body temperature.

We observe no evidence of time dependence of the characteristic length  $l$ . Based on the viscoelastic character [22] of the cement lines between osteons it was suggested by one of the authors [15] that the characteristic length might be an increasing function of time. Such an effect may in this study be masked by scattering of experimental results or be too small to be seen in tests spanning only a few decades of time scale. Recently, one of the authors has reported evidence of couple stress at a frequency many times the effective frequencies used in this study [23]. At 32.7 kHz,  $l \approx 0.09$  mm, which suggests a time/frequency dependence of  $l$ .

The analytical solution used in the analysis of the present experimental results is based on isotropic theory. Human compact bone is anisotropic; it exhibits hexagonal symmetry and the plane of transverse isotropy is perpendicular to the osteon axis. In this study, the osteon axis is parallel to the specimen axis. Measurement of the effective shear modulus of a classically elastic solid in this configuration yields one of the five elements of the modulus tensor. We expect that the characteristic length determined in this study is one of several such lengths which would be required to fully characterize bone as a couple stress elastic solid. The additional characteristic length or lengths may also have a microstructural counterpart, perhaps the length of osteons.

It is also appropriate to explore the possible role of anisotropy and other factors in generating artifactual results. Lekhnitskii [24] has solved many problems in the theory of anisotropic classical elasticity. For example, in the torsion of a rod with general elastic anisotropy, which includes a misaligned hexagonal material as a special case, no size effects are predicted [24]. A parasitic bending deformation in response to applied twisting moment is, however, predicted to occur in a rod with general anisotropy. The apparatus used in this study was designed to reject any contribution of parasitic

bending to the measurement of torsional deformation. Parasitic bending was found to be negligible, which confirms the stated parallelism of specimen and material axes. Anisotropy, therefore, cannot mimic couple stress elasticity in these experiments, and is not a source of artifact. Nonlinearity is rejected as a source of artifact since maximum strains used did not exceed the proportional limit. Viscoelasticity is decoupled from couple-stress elasticity by the use of isochronals and hence cannot be a source of artifact. Gradients of hardness or other mechanical properties across the cortex have been considered as possible sources of artifact; it is concluded that the influence of these effects is small [23].

The present results do not discriminate between couple stress elastic behavior [4] and micropolar elastic behavior [7]. For the geometry used in this study [10], the more general micropolar theory predicts behavior which is essentially the same as that predicted by couple stress theory over a considerable range of the four isotropic micropolar elastic constants.

It must be emphasized that the Mindlin-Tiersten couple stress theory, like classical elasticity, is purely phenomenological, and makes reference neither to underlying cause nor to structure. A study of mechanisms would complement the present phenomenological work. We anticipate microstructural considerations, e.g., the osteonal architecture and the viscous/compliant nature of the cement line, to be significant mechanisms for the observed nonclassical behavior. One could argue that this behavior is caused by osteons rather than couple stress; we do not regard such an argument to be cogent since it involves improper mixing of two levels of description. In the same vein, the classical elasticity of ordinary materials is caused by the electrostatic interaction between atoms. One does not reject elasticity theory because the cause of the behavior is known; instead, one uses it in practical situations for which the atomistic description is excessively cumbersome.

The potential significance of these results is that couple stress effects are predicted to perturb the stress concentration around holes [4] and the stress level near interfaces with a harder material [5]. Such holes and interfaces are routinely generated during orthopaedic surgery upon bone and in the fitting of endoprotheses. Bone-prosthesis systems are now analyzed on the basis of classical elasticity, with the aim of minimizing the likelihood of prosthesis failure, loosening and of bone resorption. The question of whether couple-stress effects are sufficiently large to merit their inclusion in such stress analyses is a subject for future research.

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