Static and dynamic effects of chirality in dielectric media

R. S. Lakes
Department of Engineering Physics, Department of Materials Science,
University of Wisconsin, 1500 Engineering Drive, Madison, WI 53706-1687
lakes@engr.wisc.edu

January 31, 2017

adapted from R. S. Lakes, Modern Physics Letters B, 30 (24) 1650319 (9 pages) (2016).

Abstract
Chiral dielectrics are considered from the perspective of continuum representations of spatial heterogeneity. Static effects in isotropic chiral dielectrics are predicted, provided the electric field has nonzero third spatial derivatives. The effects are compared with static chiral phenomena in Cosserat elastic materials which obey generalized continuum constitutive equations. Dynamic monopole - like magnetic induction is predicted in chiral dielectric media.

keywords: chirality, dielectric, Cosserat

1 Introduction

Chirality is well known in electromagnetics [1]; it gives rise to optical activity in which left or right handed circularly polarized waves propagate at different velocities. Known effects are dynamic only; there is considered to be no static effect [2]. The constitutive equations for a directionally isotropic chiral material are [3], [4].

$$\mathbf{D} = k\mathbf{E} - g\frac{\partial \mathbf{H}}{\partial t} \tag{1}$$

$$\mathbf{B} = \mu\mathbf{H} + g\frac{\partial \mathbf{E}}{\partial t} \tag{2}$$

in which $\mathbf{E}$ is electric field, $\mathbf{D}$ is electric displacement, $\mathbf{B}$ is magnetic field, $\mathbf{H}$ is magnetic induction, k is the dielectric permittivity, μ is magnetic permeability and g is a measure of the chirality. Optical rotation of polarized light of wavelength λ by an angle Φ (in radians per meter) is given by $\Phi = (2\pi/\lambda)^2 cg$ with c as the speed of light. The quantity g embodies the length scale of the chiral structure because cg has dimensions of length. If the chirality arises from structure in the atomic lattice, then the length scale will be of corresponding dimensions. Chirality is manifest in the relation, Eq. 1, between electric field and electric displacement, not in Maxwell's equations Eq. 3 - 6; ρ_E is electric charge density and $\mathbf{J}_E$ is electric current density. Because g multiplies a time differentiated field quantity, effects are dynamic.

On a fine scale, one can visualize a chiral medium as containing helical or screw shaped inclusions (Fig. 1) such that a change in electric field gives rise to a spiral conduction or displacement current which via Eq. 6 gives rise to a magnetic field.

$$div\mathbf{D} = 4\pi\rho_E \tag{3}$$

$$curl\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{4}$$

$$div\mathbf{B} = 0 \tag{5}$$

$$curl\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J}_E \tag{6}$$

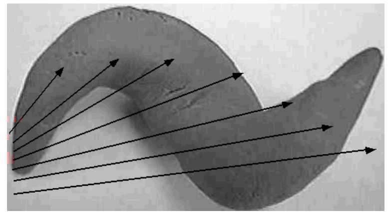

Figure 1: Chiral inclusion in a gradient of field (arrows).

The effect of chirality in electromagnetism is therefore considered to be intrinsically dynamic. The best known manifestation of this chirality is the rotation of the plane of polarization of light: optical activity [5 6]. Usually the physical origin of the behavior is asymmetry on the atomic or molecular scale, however, chiral thin films made of nanometer scale helical columns were designed to exhibit controllable optical rotation [7].

We develop in this letter a framework by which the phenomenon of static chirality can occur in dielectric solids and the relation to chirality in elastic solids. Also dynamic magnetic effects, specifically monopole-like fields, are demonstrated to be possible in chiral dielectrics.

2 Static chirality in dielectrics

Static chirality is considered in this section; chirality in electromagnetic media is usually considered to be dynamic. The reason is that chirality has no effect on second rank tensor properties such as the dielectric tensor or the elasticity tensor. To demonstrate this, consider the tensorial dielectric equation

$$D_i = k_{ij} E_j \tag{7}$$

in which k_{ij} is the usual dielectric permittivity.

The transformation law for the permittivity tensor under coordinate changes is

$$k'_{ij} = a_{im} a_{jn} k_{mn} = \frac{\partial x_m}{\partial x'_i} \frac{\partial x_n}{\partial x'_j} k_{mn} \tag{8}$$

A chiral material is sensitive to an inversion of coordinates. For an inversion, the transformation matrix is just the negative of a Kronecker delta

$$a_{im} = -\delta_{im} \tag{9}$$

$$k'_{ij} = (-1)\delta_{im}(-1)\delta_{jn} k_{mn} = (-1)^2 k_{ij} = k_{ij} \tag{10}$$

So the dielectric permittivity is unchanged by chirality. Similarly, other material properties, such as density or thermal expansion or the classical elastic modulus tensor, which are describable by tensors of even rank, are unchanged by chirality.

Tensor properties of odd rank are zero if there is inversion symmetry because they change sign under an inversion of coordinates. They can be nonzero only if there is chirality. For example piezoelectricity is governed by a third rank tensor, and strain gradient elastic theories are governed by a fifth rank tensor. Piezoelectric solids must therefore be asymmetric with respect to inversion of the coordinates.

A dielectric with chiral microstructure may be regarded as nonlocal. The reason is that a size scale is associated with chirality. Nonlocality may be expressed as follows.

$$D_i(\mathbf{r}) = \int k_{ij}(\mathbf{r} - \mathbf{r}') E_j(\mathbf{r}') \mathbf{dr}' \tag{11}$$

in which the dielectric sensitivity k_{ij} is function of position. It functions as a kernel which provides sensitivity to the field in a region of space denoted by spatial coordinates $\mathbf{r}'$. Therefore the constitutive equation for polarization is modified by writing an expansion in gradient terms in electric field E_j in which the comma denotes differentiation with respect to the variable represented by the following index.

$$D_i = k_{ij}E_j + k_{ijk}E_{j,k} + k_{ijkl}E_{j,kl} + k_{ijklm}E_{j,klm} + \cdots \tag{12}$$

Here k_{ij} is the usual dielectric permittivity and k_{ijk} is a third rank tensor representing the sensitivity to gradient effects and with units of permittivity times length. The tensors k_{ijkl} and k_{ijklm} describe sensitivity to higher gradients in field. Similarly for magnetism,

$$B_i = \mu_{ij}H_j + \mu_{ijk}H_{j,k} + \mu_{ijkl}H_{j,kl} + \mu_{ijklm}H_{j,klm} + \cdots \tag{13}$$

As for third rank sensitivity, in a directionally isotropic chiral solid, all third rank sensitivity elements are said to vanish [8]. Indeed, for the examples given, (piezoelectricity and the Pockels electro-optic effect), there is coupling with a symmetric tensor of second rank, stress in the case of piezoelectricity. There is, however, one isotropic third rank tensor, the permutation symbol. Because it is antisymmetric, there is no coupling with symmetric properties of second rank. An exception can occur [9] in Cosserat elastic solids for which the stress is asymmetric. As for dielectric materials, there is no requirement of symmetry of third rank indices in Eq. 12 so nonzero coupling is possible in the isotropic case. However the second term in Eq. 12 is proportional to curl $\mathbf{E}$ which gives $\frac{\partial \mathbf{B}}{\partial t}$ via Eq. 4. So the gradient coupling via an isotropic third rank sensitivity provides a dynamic chiral effect as anticipated by Eq. 1.

The form of third rank sensitivity differs in anisotropic solids. For a cubic material such as a crystal, third rank terms need not be antisymmetric. Specifically, [13] a third rank property such as piezoelectricity obeys, in the cubic system, $d_{123} = d_{213} = d_{312}$ (in the reduced notation, $d_{14} = d_{25} = d_{36}$) with all other elements zero. For this case $E_1 = k_{123}E_{2,3} + k_{132}E_{3,2} = k(E_{2,3} + E_{3,2})$. The corresponding form in magnetism $B_1 = \mu(H_{2,3} + H_{3,2})$, which resembles a strain in mechanics, is in contrast to curl H, for which the corresponding term is $(H_{2,3} - H_{3,2})$. A lower symmetry is required to achieve nonzero k_{111} or μ_{111} . Elements of that type provide sensitivity to the sort of gradient that occurs in a cone subjected to an axial field.

Fourth rank elements, as seen above, are unchanged in the presence of chirality. So, static chiral effects in isotropic chiral materials cannot involve a fourth rank coupling tensor.

Fifth rank coupling is possible. This linear constitutive equation is distinct from fifth rank nonlinearities [10] in chiral dielectrics. For isotropic chiral liquids [11], the fifth rank tensor in Eq. 12 was expressed (with k_{chiral} as a number describing the strength of chirality and e_{jlm} as the permutation symbol)

$$k_{ijklm} = k_{chiral}(\delta_{ij}e_{jlm} + \delta_{il}e_{jkm} + \delta_{jk}e_{ilm} + \delta_{jl}e_{ikm}) \tag{14}$$

The most general fifth rank tensor [12] has ten terms:

$$\begin{aligned} k_{ijklm} = (&C_1\delta_{lm}e_{ijk} + C_2\delta_{km}e_{ijl} + C_3\delta_{kl}e_{ijm} + C_4\delta_{jm}e_{ikl} \\ &+C_5\delta_{jl}e_{ikm} + C_6\delta_{jk}e_{ilm} + C_7\delta_{im}e_{jkl} + C_8\delta_{il}e_{jkm} \\ &+C_9\delta_{ik}e_{jlm} + C_{10}\delta_{ij}e_{klm}) \end{aligned} \tag{15}$$

Owing to the form of this isotropic tensor, there must be gradients in several directions to achieve an effect via Eq. 12. Chirality manifests itself in a directionally isotropic medium in first time derivatives of the field and in third but not in the first spatial derivatives of the field.

Units of the fifth rank tensor are permittivity times the cube of length. The length scale in the continuum representation is expected to be related to the structural length scale of the chiral inclusions (Fig. 1). The characteristic length is usually on the same order as the size of the chiral structural elements. These structural elements are much smaller than the specimen size for a continuum view of a structured medium to be sensible. Therefore static chiral effects in the isotropic case are expected to be weak unless the inclusions are designed to produce a strong effect. An experimental test might be done in the vicinity of a conductor so that the field is weak but the gradient is large. Specifically, a re-entrant (concave) corner in a conductive medium provides the potential for a null experiment because the field near the corner should tend to zero for a non-chiral dielectric, but could be non-zero in the presence of chirality. Moreover there are field gradient components in several directions. A capacitor with tilted plates would not suffice as an experimental probe because the gradient in field is in only one direction.

3 Static chirality in elasticity

As for a comparison with elasticity, chirality has no mechanical effect in classical elasticity which is governed by a fourth rank tensor, the elastic modulus: C_{klij} is the usual elastic modulus tensor, ϵ_{ij} is the strain and σ_{kl} is the stress.

$$\sigma_{kl} = C_{klij}\epsilon_{ij} \tag{16}$$

The modulus tensor is of even rank, therefore it is unchanged by an inversion of coordinates. One can envisage a gradient elasticity relation between stress σ_{kl} and strain ϵ_{ij},

$$\sigma_{kl} = C_{klij}\epsilon_{ij} + C_{klijm}\epsilon_{ij,m} \tag{17}$$

with a fifth rank elastic modulus tensor C_{klijm} which can incorporate chirality. As in the case of dielectric media, the gradient term can be interpreted as the second term of an expansion of a general nonlocal relationship between cause and effect. Chirality has been analyzed in the context of chiral Cosserat elasticity which allows a rotational degree of freedom ϕ_k of points as well as the usual translation, and a distributed torque per area m_{kl}. Both Cosserat elasticity and classical elasticity are continuum representations of materials with microstructure; Cosserat elasticity incorporates more freedom. Chiral elastic materials are also called hemitropic, noncentrosymmetric or acentric. For a chiral material isotropic with respect to direction, the constitutive equations are as follows[15]. Here λ (no relation to wavelength; the same symbol is used by convention) and G are classical elastic constants, α, β, γ allow sensitivity to strain gradients, κ governs the degree of coupling between fields, and C_1, C_2, C_3 allow chirality. Certain combinations of constants have dimensions of length; the characteristic length for torsion is $\ell_t = (\frac{\beta+\gamma}{2G})^{1/2}$.

$$\sigma_{kl} = \lambda\epsilon_{rr}\delta_{kl} + 2G\epsilon_{kl} + \kappa e_{klm}(r_m - \phi_m) + C_1\phi_{r,r}\delta_{kl} + C_2\phi_{k,l} + C_3\phi_{l,k} \tag{18}$$

$$m_{kl} = \alpha\phi_{r,r}\delta_{kl} + \beta\phi_{k,l} + \gamma\phi_{l,k} + C_1\epsilon_{rr}\delta_{kl} + (C_2 + C_3)\epsilon_{kl} + (C_3 - C_2)e_{klm}(r_m - \phi_m) \tag{19}$$

Chiral elastic materials can exhibit dynamic effects such as acoustic activity in which the plane of polarization of shear waves rotates[16 17]. This is analogous to optical activity in chiral dielectric materials.

Static effects are known in chiral elastic solids. Such a material is predicted to twist when subjected to a tension force, a static effect. In a nonchiral elastic rod, strain due to a tension force is uniform. By contrast, the deformation field in a chiral elastic rod stretched by constant axial stress becomes three dimensional as governed by the boundary conditions on the local rotation field at the free surface. The rod twists in response to tension load. The twisting deformation response to uniform applied stress entails a strain gradient. Effects of Cosserat characteristic length scales[18] and stretch twist coupling occur in elastic media with structure on a scale from 0.2 mm to 2 mm in bone[19] and on a scale of centimeters in a designed chiral polymer lattice[20]. Size effects have also been predicted to occur in composite networks of fibers[21]. A chiral dielectric differs from a chiral elastic solid in that an an applied static uniform electric field does not generate gradients in the response field.

Chirality in both elastic and electromagnetic media can be manifest as rotation of polarized waves, and in both cases the effect is linked to a characteristic length associated with the microstructure (Fig. 1). Static effects due to chirality can occur in both elasticity and electromagnetism provided there are spatial gradients of the fields.

4 Coupling with magnetism

Dirac magnetic monopoles have never been observed. If they were to exist, a monopole density ρ_M would give rise to a magnetic field via Eq. 5 modified to become div $\mathbf{B} = 4\pi\rho_M$. A corresponding magnetic current $\mathbf{J}_M$ would appear in Eq. 4. A mathematical link between chirality and magnetic monopoles[22] has been suggested in the context of equivalent descriptions of chiral behavior. Chirality can give rise to monopole - like dynamic fields as shown in the following.

Envisage a spherical capacitor (Fig. 2) filled with chiral dielectric. Suppose that a signal generator creates a radial electric field sinusoidal in time; Eq. 2 predicts that a radial monopole-like magnetic induction results.

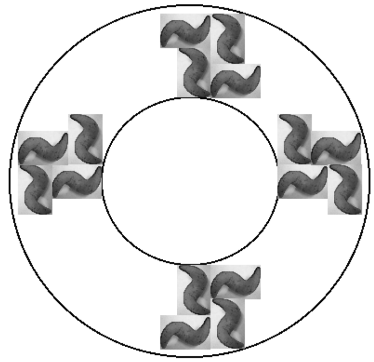

Figure 2: A spherical capacitor filled with a material containing chiral inclusions.

Specifically, from Eq. 2,

$$div\mathbf{B} = div\mu\mathbf{H} + g div\frac{\partial \mathbf{E}}{\partial t} \tag{20}$$

Combining with Eq. 1, and incorporating Eq. 5, that magnetic monopoles are not observed,

$$div\mathbf{H} = \frac{1}{\mu}[-g\frac{\partial}{\partial t}div\frac{\mathbf{D}}{k} - g\frac{\partial}{\partial t}div(\frac{g}{k}\frac{\partial \mathbf{H}}{\partial t})] \tag{21}$$

With Eq. 3, charge density ρ_E related to electric displacement,

$$div\mathbf{H} + \frac{g^2}{\mu k}\frac{\partial^2}{\partial t^2}div\mathbf{H} = -\frac{4\pi g}{\mu k}\frac{\partial \rho_E}{\partial t} \tag{22}$$

So, a time rate of change in electric charge density (e.g. on the inner sphere) acts as a source of divergence of magnetic induction **H**. In the spherical capacitor case **H** will be radial and will thus resemble the field of a monopole. Unlike a true monopole, the effect is purely dynamic. Moreover, since the charges on the inner and outer conductors of the capacitor are equal and opposite, the source term vanishes outside the capacitor. This is understandable in view of the fact that Maxwellian electromagnetism does not support the longitudinal waves which would result if a time-varying radial magnetic field extended into free space. Even so, the outer capacitor conductor can become sufficiently remote that monopole - like fields may be observed in a large volume of space. Also, solution of Eq. 22 for a region without charge density gives rise to resonance effects with natural angular frequency $\sqrt{\mu k}/\ g$. One may envisage g as a wave travel time across the chiral inclusion, but the inclusion itself may have resonant properties as in meta-materials [23 24],[25]. In such materials, enhanced effects are possible.

5 Discussion

Chirality has traditionally been considered at the molecular scale. Chirality can also be expressed at larger scales, for example tens of microns in composites [7] or centimeters in 3D printed chiral polymer lattice [20] or in designed broadband circular polarizers based on metamaterials [26]. Consequently, effects associated with nonlocality can appear on a macrosopic scale. Nonlocality [27] is pertinent to dielectric media; nonlocality can be expressed in terms of an expansion of higher gradients of the independent variable. Nonlocal effects are known in piezoelectric materials [28], which are always chiral. Nonlocality in piezoelectric materials gives rise to a coupling [29] between electric polarization and *bending*, in addition to the usual coupling with uniform deformation. The effect increases in magnitude as one approaches a phase transformation in a ferroelectric. More recently such effects in which there is coupling between mechanical strain gradient and electric polarization, have been referred to as flexoelectric [30]. Gradient effects are of interest because they can lead to more intense response in slender transducer elements made of ferroelectric materials and because they can result in a smearing of phase transitions[31].

The predicted monopole-like effects are dynamic and are distinct from the Dirac monopole which is expected to be a static elementary particle but has not been observed. Monopole-like entities, considered[32] as excitations that manifest like point charges, have been observed in spin ice[33], a class of frustrated magnetic material. Spin ice states are thought to contain arrangements of aligned magnetic dipoles that resemble solenoidal tubes. The ends of the tubes resemble magnetic monopoles. Such effects require cryogenic temperatures, in contrast to Dirac monopoles and the dynamic effects studied here.

6 Conclusion

In conclusion, this letter presents new effects in chiral dielectric media, specifically static polarization in response to third spatial gradients of the electric field. Such effects are analogous to gradient sensitivity found in elastic solids described by generalized continua. Dynamic monopole-like fields of magnetic induction are predicted in chiral dielectrics.

7 Acknowledgments

We gratefully acknowledge support of this research by the National Science Foundation via Grant CMMI-1361832 and from ACS-PRF.

References

[1] E. J. Post, *Formal structure of electromagnetics*, North Holland, Amsterdam, (1962).

[2] E. O. Kamenetskii, Quasistatic principles in the macroscopic electrodynamics of bianisotropic media *Phys. Rev. E* **58**, (1998) 7965–7973.

[3] E. U. Condon, Theories of optical rotatory power, *Rev. Mod. Physics* **9**, (1937) 432-457.

[4] M. P. Silverman, Reflection and refraction at the surface of a chiral medium: comparison of gyrotropic constitutive relations invariant under a duality transformation, *J. Optical Soc. Am. A*, **3**, (1986) 830-837.

[5] T. M. Lowry, *Optical Rotatory Power*, Longmans, Green, London, (1935).

[6] L. D. Barron, *Molecular light scattering and optical activity*, Cambridge University Press, Cambridge, (1982).

[7] K. Robbie, M. J. Brett, and A. Lakhtakia, Chiral sculpted thin films, *Nature*, **384**, (1996) 616.

[8] D. R. Lovett, *Tensor Properties of Crystals,* A. Hilger, Phila. PA, (1989), p. 123.

[9] R. S. Lakes, Third-rank piezoelectricity in isotropic chiral solids, *Appl. Phys. Lett.*, **106**, May (2015) 212905.

[10] A. P. Shkurinov, A. V. Dubrovskii, and N. I. Koroteev, Second harmonic generation in an optically active liquid: Experimental observation of a fourth-order optical nonlinearity due to molecular chirality *Phys. Rev. Lett.* **70**, (1993) 1085-1088.

[11] H. R. Brand and H. Pleiner, Macroscopic dynamics of an isotropic liquid phase containing chiral objects, *Phys. Rev. A* **46**, (1992) R3004-R3007.

[12] A. S. J. Suiker and C. S. Chang, Application of higher-order tensor theory for formulating enhanced continuum models, *Acta Mechanica* **142** (2000), 223-234

[13] J. F. Nye, *Physical Properties of Crystals.* Oxford University Press, Oxford, (1957).

[14] Y. Weitsman, Initial stresses and skin effects in a hemitropic Cosserat continuum, *J. Appl. Mech.* **34**, (1967) 160-164.

[15] R. S. Lakes and R. L. Benedict, Noncentrosymmetry in micropolar elasticity, *Int. J. Engng Sci.*, **29**, (1982) 1161-1167.

[16] Varadan, V.V. Lakhtakia, A. and Varadan, V.K. Geometry can be the basis for acoustic activity (a la optical activity) in composite media, *Journal of Wave-Material Interaction*, **1**, (1986) 315 - 323.

[17] S. K. Yang, V. V. Varadan, and V. K. Varadan, Acoustical activity in structurally chiral composites, *J. Acoust. Soc. Am.* **85**, (1989) S116-S116.

[18] R. S. Lakes, Experimental microelasticity of two porous solids, *International Journal of Solids and Structures*, **22** (1986) 55-63.

[19] R. S. Lakes, Is bone elastically noncentrosymmetric? *Proceedings 34th ACEMB*, (1981) p. 286, Houston, Sept. (1981).

[20] C. S. Ha, M. E. Plesha, R. S. Lakes, *Chiral three-dimensional isotropic lattices with negative Poisson's ratio*, *Physica Status Solidi B*, **253**, (7), (2016) 1243-1251.

[21] H. Wada and Y. Tanaka, Mechanics and size-dependent elasticity of composite networks, *EPL*, **87**, (2009) 58001.

[22] A. Lakhtakia, A case for magnetic sources, *Physics Essays,* **4**, (1991) 105-108.

[23] S., C. Linden, M. Enkrich, J. Wegener, T. Zhou, Koschny, and C. M. Soukoulis, Magnetic Response of Metamaterials at 100 Terahertz, *Science* **306**: (2004) 1351-1353.

[24] J. B. Pendry, A Chiral Route to Negative Refraction, *Science*, **306**: (2004) 1353-1355.

[25] N. A. Nicorovici and G. W. Milton, On the cloaking effects associated with anomalous localized resonance , *Proceedings of the Royal Society of London, A*, **462**, (2006) 3027 - 3059.

[26] J. K. Gansel, M. Thiel, M. S. Rill, M. Decker, K.Bade, V. Saile, G. von Freymann, S. Linden, M. Wegener, Gold helix photonic metamaterial as broadband circular polarizer, *Science* **325**, (2009) 1513-1515.

[27] D. Lu, W. Hu, Q. Guo, The relation between optical beam propagation in free space and in strongly nonlocal nonlinear media, *EPL* **86**, (2009) 44004.

[28] E. V. Bursian and N. N. Trunov, Nonlocal piezoelectric effect, *Sov. Physics Solid State,* **16** (4) (1974) 760-762.

[29] E. V. Bursian and O. I. Zaikovskii, Changes in the curvature of a ferroelectric film due to polarization, *Sov. Physics Solid State*, **10** (5) (1968) 1121-1124.

[30] W. Ma and L. E. Cross, Flexoelectricity of barium titanate, *Appl. Phys. Lett.* **88**, (2006) 232902.

[31] A. M. Bratkovsky, and A. P. Levanyuk, Smearing of Phase Transition due to a Surface Effect or a Bulk Inhomogeneity in Ferroelectric Nanostructures, *Phys. Rev. Lett.* **94**, (2005) 107601.

[32] C. Castelnovo, R. Moessner and S. L. Sondhi, Magnetic monopoles in spin ice, *Nature*, **451**, (2007), 42-45.

[33] S. T. Bramwell, S. R. Giblin, S. Calder, R. Aldus, D. Prabhakaran, and T. Fennell, Measurement of the charge and current of magnetic monopoles in spin ice, *Nature*, **461** (2009), 956-959.